MATH 21

College Algebra 1 Lecture Notes

Factoring and Foiling

1. $(a+b)^2 = a^2 + 2ab + b^2$.
2. $(a-b)^2 = a^2 - 2ab + b^2$.
3. $(a+b)(a-b) = a^2 - b^2$.
4. $(a+b)(x+y) = ax + ay + bx + by$.
5. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$.
6. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

Factor completely.

1. $2t^2 - 8$

2. $4x^2 + 4$

3. $n^2 + 18n + 77$

4. $x^2 - 3x - 54$

5. $t^2 + 18t + 81$

6. $36a^2 - 12a + 1$

7. $6b^2 + 13b + 6$

8. $x^2 - (y+2)^2$

9. $x^5 - x$

10. $3xy + 15x - 2y - 10$

Properties of Rational Expressions

1. You can't divide by 0. 2. $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$. 3. $\frac{-a}{-b} = \frac{a}{b}$. 4. $\frac{a \cdot k}{b \cdot k} = \frac{a}{b}$.

A ***rational expression*** is a fraction consisting of two polynomials.

Simplify each of the following expressions.

1. $\frac{12}{16}$

2. $\frac{30}{42}$

3. $\frac{32}{80}$

4. $\frac{6x}{15}$

5. $\frac{14xy^2}{12y}$

6. $\frac{x^2 - 9}{x^2 + 3x}$

7. $\frac{a^3 + 7a^2 + 10a}{a^3 - 7a^2 - 18a}$

8. $\frac{3n^2 + 16n - 12}{7n^2 + 44n + 12}$

9. $\frac{n^2 - 36}{6 - n}$

Rules for Multiplying and Dividing Rational Expressions

1. $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} = \frac{ac}{bd}$.
2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$.

Perform the indicated operations and simplify.

1. $\frac{2}{3} \cdot \frac{4}{5}$

2. $\frac{12}{9} \cdot \frac{4}{18}$

3. $\frac{x^2}{y} \cdot \frac{y^2}{x}$

4. $\frac{x+2}{x-3} \cdot \frac{x^2-6x+9}{x^2-4}$

5. $\frac{5a^2+20a}{a^3-2a^2} \cdot \frac{a^2-a-12}{a^2-16}$

6. $\frac{2}{3} \div \frac{4}{5}$

7. $\frac{x-6}{x^2-2x+1} \div \frac{x^2-3x-18}{x-1}$

8. $\frac{9n^2-12n+4}{n^2-4n-32} \div \frac{3n^3-2n^2}{n^2+4n}$

9. $\frac{x^2-3xy+2y^2}{x^2y-xy^2} \div \frac{x-2y}{x^3-xy^2}$

10. $\frac{x^4-16}{x+2} \div \frac{x^2+4}{x+5}$

Rules for Adding and Subtracting Rational Expressions

1. $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.
2. $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$.
3. **Don't** "cancel" anything that is not being multiplied.

The ***least common denominator*** of two fractions is the smallest quantity into which both denominators divide evenly.

Perform the indicated operations and simplify.

1. $\frac{4}{7} + \frac{2}{7}$

2. $\frac{3}{8} + \frac{7}{10}$

3. $\frac{1}{2} - \frac{1}{6} + \frac{1}{5}$

4. $\frac{5-x}{2} + \frac{2x}{3}$

5. $\frac{7}{6x} - \frac{4}{3y}$

6. $\frac{x}{4} - \frac{4}{x}$

7. $\frac{4}{2x-3} - \frac{2}{7x}$

8. $\frac{2}{x} + \frac{3}{x+4}$

9. $\frac{4}{4+x} - \frac{2}{2+x}$

10. $\frac{4}{2x-3} - \frac{5x}{3-2x}$

11. $\frac{3}{2x+1} + \frac{2}{2x+4}$

12. $\dfrac{3x}{x-4} - 2$

13. $\frac{2x+12}{x+2} - 2\left(\frac{3+x}{x-1}\right)$

14. $\frac{7}{3x^2} - \frac{9}{4x} - \frac{5}{2x}$

15. $\frac{4}{x^2+1} - \frac{4}{x^2}$

16. $\frac{3x+1}{3x+2} - \frac{3x+2}{3x+1}$

17. $\frac{3x}{4x-1} - \frac{7x-1}{4x-1}$

Steps for Simplifying Fractions
Step 1. Factor the denominators.
Step 2. Find the LCD.
Step 3. Get the common denominator under both fractions.
Step 4. Combine the numerators.
Step 5. Simplify.
Step 6. Look for ways to simplify by factoring.

Perform the indicated operations and simplify.

1. $\frac{3x}{x^2-6x} + \frac{4}{x}$

2. $\frac{4a-4}{a^2-4} - \frac{3}{a+2}$

3. $\frac{3n}{n^2-36} - \frac{2}{5n+30}$

4. $\frac{5}{x^2-1} + \frac{9}{x^2+2x+1}$

5. $\frac{6}{b^2-3b-54} - \frac{10}{b^2+5b-6}$

6. $\frac{3}{x+1} + \frac{x+5}{x^2-1} - \frac{3}{x-1}$

7. $\frac{t-3}{2t+1} + \frac{2t^2+19t-46}{2t^2-9t-5} - \frac{t+4}{t-5}$

Complex fractions are fractions that have fractions in the numerator or denominator.
Technique 1. Combine fractions in the numerator and denominator separately.
Technique 2. Multiply numerator and denominator by their simultaneous LCD.

8. $\dfrac{\frac{6a^2}{b}}{\frac{ab}{4}}$

9. $\dfrac{\frac{3}{8}+\frac{3}{4}}{\frac{5}{8}-\frac{7}{12}}$

10. $\dfrac{\frac{4}{xy}-\frac{3}{y^2}}{\frac{1}{x}+\frac{3}{y}}$

11. $\dfrac{\frac{2}{x-3}-\frac{3}{x+3}}{\frac{5}{x^2-9}-\frac{2}{x-3}}$

12. $1-\dfrac{a}{\frac{1}{a}+4}$

13. $\dfrac{\frac{3}{n-5}-2}{1-\frac{4}{n-5}}$

Steps for Solving Fractional Equations
Step 1. Find the LCD.
Step 2. Multiply both sides by the LCD.
Step 3. Gather like terms.
Step 4. If necessary, get 0 on one side of the equation and factor the other side. Then set the individual factors equal to zero and solve.
Step 5. Check to make sure your solution satisfies the original equation.

Solve each equation.

1. $\frac{x+2}{5} + \frac{x-1}{6} = \frac{3}{5}$

2. $\frac{3}{n} + \frac{1}{6} = \frac{11}{3n}$

3. $\frac{x+2}{5} + \frac{x-1}{6} = \frac{3}{5}$

4. $\frac{3}{2x-1} = \frac{5}{3x+2}$

5. $n - \frac{3}{n} = \frac{26}{3}$

6. $\frac{x+6}{27} = \frac{1}{x}$

7. $\frac{x}{x-4} - \frac{2}{x+3} = \frac{20}{x^2-x-12}$

Ratios and Proportions
A ***ratio*** is a fraction used to compare two numbers.
A ***proportion*** is a statement that two ratios are equal.
Cross Multiplication. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

8. The sum of a number and its reciprocal is $\frac{25}{12}$. Find the number.

9. This class has 48 students. The ratio of men to women is 5 to 7. How many women are in this class?

10. The sum of two numbers is 84. The ratio of the larger number to the smaller number is 4 to 3. What are the two numbers?

11. The sum of two numbers is 28. When the larger number is divided by the smaller number, the quotient is 2 and the remainder is 4. What is the larger number? (*Hint:* $\frac{\text{Divident}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$.)

12. An inheritance of \$300,000 is to be divided between a son and the local heart fund in the ratio of 3 to 1. How much money will the son receive?

13. My (rectangular) desk has a perimeter of 14 feet. If the ratio of its width to its length is 2 to 5, then what are its dimensions?

Steps for Solving Fractional Equations
Step 1. Find the LCD.
Step 2. Multiply both sides by the LCD.
Step 3. Gather like terms.
Step 4. If necessary, get 0 on one side of the equation and factor the other side. Then set the individual factors equal to zero and solve.
Step 5. Check to make sure your solution satisfies the original equation.

Solve each equation.

1. $\frac{4}{t-1} + 2 = \frac{4}{t^2-t}$

2. $\frac{2n-1}{n^2-n-12} = \frac{3}{n-4} + \frac{2}{n+3}$

3. $1 + \frac{1}{n-1} = \frac{1}{n^2-n}$

4. $\frac{n}{n-5} + \frac{2}{n-6} = \frac{2}{a^2-11a+30}$

5. $\frac{5y-4}{6y^2+y-12} - \frac{2}{2y+3} = \frac{5}{3y-4}$

6. $\frac{x}{2x^2+5x} - \frac{x}{2x^2+7x+5} = \frac{2}{x^2+x}$

Steps for Solving Word Problems
Step 1. Carefully read the problem.
Step 2. Define variables.
Step 3. Set up equations.
Step 4. Solve the equation.
Step 5. Check your answer and write the answer as a sentence.
$Distance = Rate \cdot Time$

7. Solve $\frac{1}{A} = \frac{1}{B} + \frac{1}{C}$ for B.

8. Solve $\frac{x+3}{z-2} = \frac{2}{y}$ for x.

9. Harry flies his broom 96 miles in the same time it takes Luna to fly her thestral 56 miles. If the Harry flew 20 miles per hour faster than Luna, find their rates.

10. Luke can mow a lawn in 3 hours. Duke can mow the lawn in 2 hours. How long will it take them to mow the lawn if they are working together?

11. My wife (Jenny) can clean our cluttered living room in 30 minutes. My son (Andrew) can clutter up the living room in 40 minutes. If Jenny starts cleaning the cluttered living room while Andrew is busy cluttering it back up, how long will it take for the room to get cleaned?

12. It takes Nemo three times as long to clean a sea cucumber as it takes Marlin. If it takes them 1 hour to clean a sea cucumber when working together, then how long will it take Nemo working alone?

13. It takes Lester 1 hour longer than it takes Chester to paint a picture. If they paint 5 pictures in 6 hours, how many were painted by Lester?

Properties of Exponents

1. $b^m \cdot b^n = b^{m+n}$
2. $(b^m)^n = b^{mn}$
3. $(ab)^n = a^n b^n$
4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
5. $\frac{b^n}{b^m} = b^{n-m}$
6. $b^0 = 1$
7. $b^{-n} = \frac{1}{b^n}$

Simplify each of the following expressions.

1. $6^4 \cdot 6^{-2}$
2. $(3^{-2})^{-3}$
3. $(5 \cdot 3^{-2})^2$
4. $\left(\frac{6^{-1}}{4^{-2}}\right)^{-3}$
5. $\frac{2^{-3}}{2^{-5}}$
6. $x^{-1} \cdot x^3$
7. $(x^4)^5$
8. $(a^3 \cdot b^{-1})^4$
9. $\left(\frac{x^2}{y^3}\right)^2$
10. $\frac{x^{-3}}{x^{-5}}$
11. $(-4x^{-1}y^2)(6x^3y^{-4})$
12. $\frac{108a^{-5}b^{-4}}{9a^{-2}b}$
13. $\frac{-48ab^2}{-6a^3b^5}^{-2}$
14. $2^{-1} - 3^{-1}$
15. $a^{-2} + a^{-1}b^{-2}$

Radicals and n^{th} Roots
A (non-negative) number a is an n^{th} ***root*** of b if and only if $a^n = b$. We write $\sqrt[n]{b} = a$.

Simplify each of the following expressions.

1. $\sqrt[2]{16}$
2. $\sqrt[3]{125}$
3. $\sqrt[4]{81}$
4. $\sqrt[7]{128}$

Properties of Radicals
If $b, c \geq 0$, 1. $(\sqrt[n]{b})^n = b = \sqrt[n]{b^n}$ 2. $\sqrt[n]{bc} = \sqrt[n]{a}\sqrt[n]{b}$ 3. $\sqrt[n]{\frac{b}{c}} = \frac{\sqrt[n]{b}}{\sqrt[n]{c}}$

Write each expression in simplest radical form.

5. $\sqrt{18}$
6. $\sqrt[3]{54}$
7. $\sqrt{\frac{9}{36}}$
8. $\sqrt{\frac{144}{36}}$
9. $\sqrt{28}$
10. $3\sqrt[3]{24}$

Rules for Simplifying Radicals
1. No fractions can appear under the radical.
2. No radicals can be in the denominator of a fraction.
3. All exponents under the radical must be smaller than the index.

Write each expression in simplest radical form.

11. $\sqrt{\frac{2}{7}}$
12. $\frac{\sqrt{4}}{\sqrt{3}}$
13. $\frac{\sqrt{5}}{\sqrt{18}}$
14. $\frac{3\sqrt{2}}{4\sqrt{5}}$
15. $\frac{2}{\sqrt[3]{2}}$

Adding Radical Expressions
When we add radicals we treat the simplified radical much like a variable.

Simplify each of the following expressions.

1. $2\sqrt{7} - 5\sqrt{7}$

2. $3\sqrt{18} - 8\sqrt{2}$

3. $6\sqrt{12} + \sqrt{3} - 2\sqrt{48}$

4. $\sqrt{6} - \sqrt{12}$

Rules for Simplifying Radicals
1. No fractions can appear under the radical.
2. No radicals can be in the denominator of a fraction.
3. All exponents under the radical must be smaller than the index.

Write each expression in simplest radical form.

5. $\sqrt{50y}$

6. $\sqrt{36x^5y^6}$

7. $\sqrt[3]{24a^8b^9}$

8. $\frac{3}{\sqrt{12x}}$

9. $\frac{\sqrt{5y}}{\sqrt{18x^3}}$

10. $\sqrt{12x + 8y}$

11. $4\sqrt{8n} + 3\sqrt{18n} - 2\sqrt{72n}$

12. $2\sqrt{40x^5} - 3\sqrt{90x^5} + 5x\sqrt{160x^3}$

Multiplying Radicals
Recall that $\sqrt[n]{bc} = \sqrt[n]{b}\sqrt[n]{c}$.

Multiply and simplify.

1. $(3\sqrt{7})(4\sqrt{2})$

2. $(2\sqrt{12})(5\sqrt{6})$

3. $(6\sqrt{6})(-3\sqrt{8})$

4. $(\sqrt[3]{14})(\sqrt[3]{12})$

5. $\sqrt{3}(\sqrt{7}+\sqrt{10})$

6. $-2\sqrt{3}(3\sqrt{12}-9\sqrt{8})$

7. $\sqrt{2x}(\sqrt{12xy}-\sqrt{8y})$

8. $(\sqrt{7}-2)(\sqrt{7}-8)$

9. $(\sqrt{2}-\sqrt{3})(\sqrt{5}+\sqrt{7})$

10. $3\sqrt[3]{3}(4\sqrt[3]{9}+5\sqrt[3]{7})$

11. $(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})$

Conjugates and Dividing Radicals
A pair of expressions $a+b$ and $a-b$ are said to be ***conjugate***. The product of a conjugate pair gives us the difference of squares: $(a+b)(a-b) = a^2 - b^2$.

12. What is the conjugate of $3\sqrt{7x} - 4y$?

Simplify each expression.

13. $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})$

14. $\dfrac{5}{\sqrt{10} - 3}$

15. $\dfrac{\sqrt{3}}{3\sqrt{2} - 5}$

16. $\dfrac{\sqrt{6}}{2\sqrt{7} - \sqrt{2}}$

17. $\dfrac{\sqrt{x}}{\sqrt{x} - 1}$

18. $\dfrac{\sqrt{2} + \sqrt{3}}{\sqrt{3} + \sqrt{5}}$

19. True or False? $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

Solving Radical Equations
If $a = b$, then $a^n = b^n$. The converse statement does not hold in general.

Solve each equation and check your solution.

1. $\sqrt{4x} = 6$

2. $4\sqrt{x} = 3$

3. $\sqrt{x+1} + 5 = 3$

4. $\sqrt{2y-3} = 5$

5. $\sqrt{6x+5} = \sqrt{2x+10}$

6. $\sqrt{7x-6} - \sqrt{5x+2} = 0$

7. $\sqrt{x^2+3} - 2 = 0$

8. Solve the formula $r = \sqrt[3]{\frac{3}{4\pi}V}$ for V.

9. $\sqrt{x^2+2x+1} = x+3$

10. $\sqrt{n+4} = n+4$

11. $2\sqrt{x} = x-3$

12. $\sqrt{-x} - 6 = x$

13. $\sqrt[3]{3x-1} = \sqrt[3]{2-5x}$

14. $\sqrt{2x-1} - \sqrt{x+3} = 1$

15. $\sqrt{n-3} + \sqrt{n+5} = 2\sqrt{n}$

Fractional Exponents

1. $b^{\frac{1}{n}} = \sqrt[n]{b}$ 2. $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$

Simplify.

1. $16^{\frac{1}{2}}$

2. $27^{\frac{1}{3}}$

3. $(-27)^{\frac{1}{3}}$

4. $-27^{\frac{1}{3}}$

5. $8^{\frac{4}{3}}$

6. $4^{-\frac{1}{2}}$

7. $\left(\frac{27}{64}\right)^{\frac{2}{3}}$

8. Write the expression in radical form.

$(2m + 3n)^{\frac{5}{7}}$

9. Write the expression in exponent form.

$\sqrt[4]{r^3 t}$

Simplify, and leave in exponent form.

10. $(b^{\frac{2}{3}})(b^{\frac{1}{6}})$

11. $\left(2x^{\frac{1}{3}}y^{\frac{2}{5}}\right)^3$

12. $\dfrac{12x^{\frac{1}{3}}}{8x^{\frac{1}{2}}}$

Simplify, and leave in radical form.

13. $(\sqrt{2})(\sqrt[4]{2})$

14. $\dfrac{\sqrt{3}}{\sqrt[3]{9}}$

15. $\dfrac{\sqrt[3]{8}}{\sqrt[4]{4}}$

Complex Numbers
We define i to be a number such that $i^2 = -1$.
A ***complex number*** is a number of the form $a + bi$, where a and b are real numbers. We call a the ***real part*** and b the ***imaginary part***.

1. **T** **F** 7 is a complex number.

2. **T** **F** The real part of $3i$ is 0.

Adding and Subtracting Complex Numbers
To add two complex numbers we just add the real parts and imaginary parts separately. That is, we treat i as we would a variable.

Add or subtract as indicated.

3. $(3 + 2i) + (4 - 3i)$

4. $(6 - 3i) - (4 - 4i)$

Multiplying Complex Numbers
To multiply two complex numbers we treat i as we would a variable. The only difference is that we always simplify $i^2 = -1$. Note that the rule $\sqrt{ab} = \sqrt{a}\sqrt{b}$ only works when a and b are non-negative numbers.

Multiply as indicated.

5. $(3 + 2i)(4 - 3i)$

6. $(6 - 3i)(4 - 4i)$

Dividing Complex Numbers
To simplify a fraction of complex numbers, we multiply the denominator by its conjugate. Then simplify.

Divide or simplify as indicated.

7. $\dfrac{3 + 2i}{4 - 3i}$

8. $\dfrac{6 - 3i}{4 - 4i}$

9. $\dfrac{3 + i}{i}$

10. i^{19}

Quadratic Equations
A ***quadratic equation*** is an equation containing one variable with highest exponent 2. The form $ax^2 + bx + c = 0$ is called the ***standard form*** of a quadratic equation. Remember, that if $ab = 0$, then either a or b must be zero.

Solve each equation.

1. $x^2 = 15x$

2. $x^2 - 17x + 72 = 0$

3. $24x^2 + x - 10 = 0$

4. $\sqrt{x} = x - 2$

A Convenient Fact
For any real number a, the equation $x^2 = a$ is true if and only if $x = \pm\sqrt{a}$.

Solve each equation.

5. $x^2 = 16$

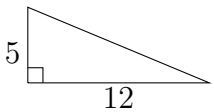
6. $7t^2 = 4$

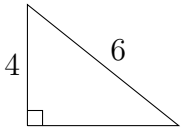
7. $(x - 3)^2 = 36$

8. $(4y + 5)^2 = 18$

Right Triangles
A right triangle with sides of length a and b and hypotenuse of length c satisfies the ***Pythagorean Theorem***: $a^2 + b^2 = c^2$.

Find the lengths of the sides of the right triangle.

9. 

10. 

11. A rectangular room is 8 feet by 15 feet. How far apart are the opposite corners?

Perfect Square Trinomial
A (monic) trinomial is a perfect square if it factors in the form $(x-b)^2$. An equation of the form $(x-b)^2 = c$ has solution $x = b \pm \sqrt{c}$. Remember that $(x+y)^2 = x^2 + 2xy + y^2$.

Fill in the blank with the number that makes the trinomial a perfect square.

1. $x^2 + 10x + ____$
2. $x^2 - 20x + ____$
3. $x^2 + 7x + ____$

Completing the Square
We can solve a quadratic equation $x^2 + bx + c = 0$ by subtracting c to the right-hand side and then completing the square. $x^2 + bx + ____ = -c + ____$. If the coefficient of x^2 is not 1, then we must first divide both sides of the equation by that coefficient.

Solve each equation.

4. $x^2 - 18 + 72 = 0$
5. $x(x-1) = 30$
6. $x^2 + 2x - 1 = 0$
7. $y^2 - 6y = -10$
8. $2x^2 + 4x = 6$
9. $4x^2 - 8x = -3$

Quadratic Formula
The solution of the quadratic equation $ax^2+bx+c=0$ is given by the ***quadratic formula***

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

Solve each equation.

1. $x^2 - 3x - 54 = 0.$

2. $2x^2 + 5x + 3 = 0$

3. $3x^2 - 2x + 5 = 0$

4. $22t^2 + 11t - 33 = 0$

The Discriminant and a Useful Check
The part under the radical $b^2 - 4ac$ is called the ***discriminant***. If $b^2 - 4ac > 0$, there are two real solutions. If $b^2 - 4ac = 0$, there is one real solution. If $b^2 - 4ac < 0$, there are two complex solutions.
Checking Sum of Roots: The two solutions of $ax^2 + bx + c = 0$ will always add up to $-\frac{b}{a}$.

Solve each equation and check the sum of the solutions.

5. $-y^2 + 7y = 4$

6. $2a^2 - 6a + 1 = 0$

Rules of Thumb
There are no set rules for choosing which method to employ to solve a quadratic equation. I suggest
1. Take a quick glance to see if it might factor easily, and if not, then
2. Use the quadratic formula.

Warning: Be sure that your solution doesn't lead to division by zero.

Solve each equation.

1. $2x^2 + 3x - 4 = 0$

2. $(x + 3)(2x + 1) = -3$

3. $\frac{2}{x} + \frac{5}{x+2} = 1$

4. $\frac{12}{t} - \frac{18}{t+8} = \frac{9}{2}$

5. $x^4 - 5x^2 + 4 = 0$

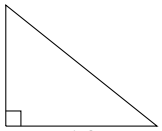
6. $x^4 - 21x^2 + 54 = 0$

7. A number plus its reciprocal is equal to 3. What is the number?

8. An 8-inch by 10-inch picture is surrounded by a frame of uniform width. The area of the picture and frame together is 120 square inches. Find the width of the frame.

9. Bart's time to travel 20 miles on his skateboard is 1 hour less than the same as Lisa's time to travel 42 miles on her bike. If Lisa's speed is 4 miles per hour faster than Bart's, then how fast did they each travel?

10. A right triangle has a base of 16 feet. The length of the hypotenuse is 8 feet more than the remaining side. What is the length of the remaining side of the triangle?



11. Ike and Mike are brothers that have square bedrooms. Mike's room is 1 foot longer than Ike's. The total area of their rooms is 221 square feet. How big is Mike's room?

12. The perimeter of a rectangle is 52 inches. The area of the rectangle is 48 square inches. What are the dimensions of the rectangle?

Intervals
To solve an inequality, we isolate the variable just like solving an equation. The only difference is, that when we multiply by a negative number, we must switch the direction of the inequality.

Solve each inequality. Write the solution set in interval form and graph its solution set on a number line.

1. $2x + 4 > 2$

2. $x - 1 < 2x - 2$

3. $-x + 6 \geq 11$

4. $\frac{5}{4}(x + 1) \leq -\frac{5}{2}$

Compound Inequalities
We use the words "and" and "or" in mathematics to form ***compound statements***. The ***intersection*** of two sets A and B (denoted $A \cap B$) is the set of all elements that are in both A and B. The ***union*** of A and B (denoted $A \cup B$) is the set of all elements in either A or B or both.

Solve each inequality. Write the solution set in interval form and graph its solution set on a number line.

5. $x > 1$ and $x < 4$

6. $x + 2 < -3$ or $x + 2 > 3$

7. $18 \leq -2x + 4 \leq 10$

8. Cosmo bowled 142 and 170 in his first two games. What must he bowl in the third game to have an average of at least 160 for the three games?

Quadratic Inequalities

We get a ***quadratic inequality*** when we replace the equal sign in a quadratic equation $ax^2 + bx + c = 0$ with an inequality sign ($>$, $<$, $\geq$, or $\leq$).

Solve each inequality and graph its solution set on a number line.

1. $(x - 3)(x - 2) > 0$

2. $x(x + 1)(x - 1) \geq 0$

3. $x^2(x - 3) \leq 0$

4. $4x^2 - x - 14 < 0$

5. $9x^2 - 6x + 1 \leq 0$

6. $2x^3 + 10x^2 > 0$

Rational Expressions and Inequalities

To solve an inequality of the form $\frac{f(x)}{g(x)} > 0$, we break up the number line into test intervals whose endpoints are the roots of $f(x)$ and $g(x)$. Rephrased, we find the solutions of $f(x) = 0$ and $g(x) = 0$, and then test the intervals in between the solutions.

Solve each inequality and graph the solution set on a number line.

7. $\frac{x-4}{x-2} > 0$

8. $\frac{x}{x-1} \leq 0$

9. $\frac{4-x}{x} \geq 0$

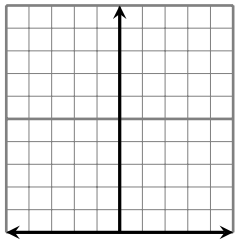
10. $\frac{x}{x-1} > 2$

11. $\frac{x+3}{x-4} \geq 1$

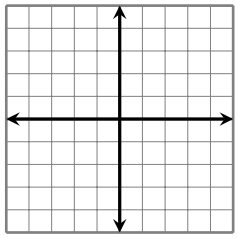
Parabolas
A parabola is the graph of an equation of the form $y = ax^2 + bx + c$. The graph of a parabola resembles a valley $\bigcup$ or an arch $\bigcap$.

Graph each parabola.

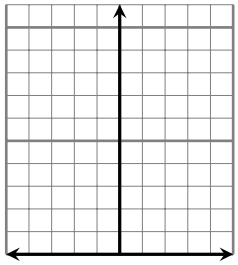
1. $y = x^2$



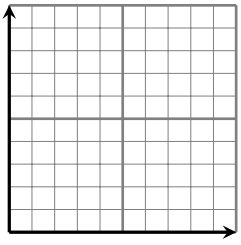
2. $y = x^2 - 4$



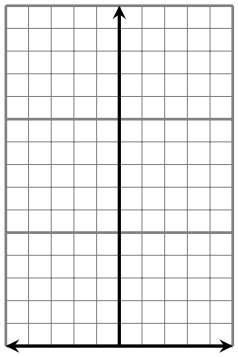
3. $y = x^2 + 2$



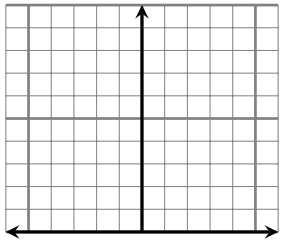
4. $y = (x - 4)^2$



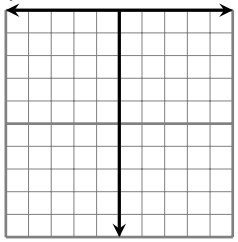
5. $y = 3x^2$



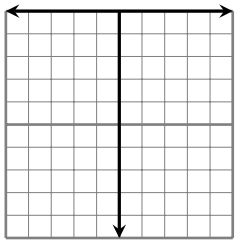
6. $y = \frac{1}{3}x^2$



7. $y = -x^2$



8. $y = -\frac{1}{2}x^2$



Graphing a Parabola

We know the following transformations of the standard parabola $y = x^2$:

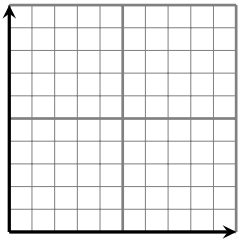
1. Horizontal Shift by h units — $y = (x - h)^2$
2. Vertical Shift by k units — $y = x^2 + k$
3. Vertical Stretch/Reflect by a factor of a — $y = ax^2$

A parabola with vertex at the point (h, k) that is a times as 'tall' as $y = x^2$ has equation

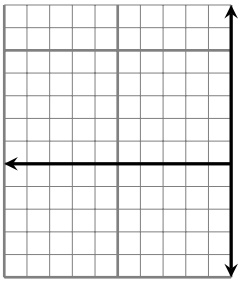
$$y = a(x - h)^2 + k.$$

Graph each parabola.

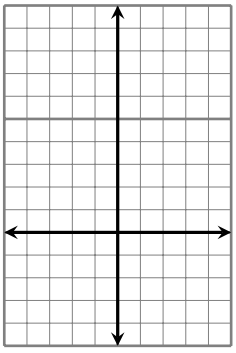
9. $y = (x - 2)^2 + 3$



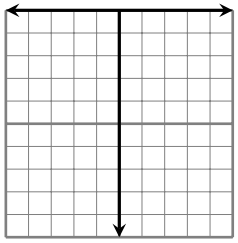
10. $y = -(x + 5)^2 - 2$



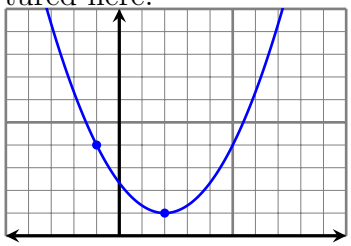
11. $y = 2x^2 - 3$



12. $y = -.5(x + 1)^2 - 1$



13. Write the equation of the parabola pictured here.



14. How does the graph of $y = 4x^2$ compare to the graph of $y = 2x^2$?

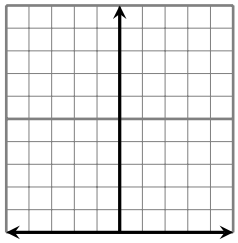
15. Will the parabolas $y = \frac{1}{4}(x - 3)^2 + 2$ and $y = -(x + 4)^2 + 4$ intersect?

Parabolas

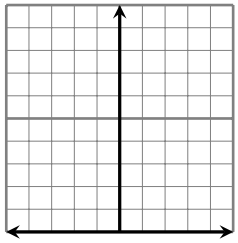
The general approach to graphing a parabola of the form $y = ax^2 + bx + c$ is to convert it to the form $y = a(x - h)^2 + k$ by completing the square.

Graph each parabola.

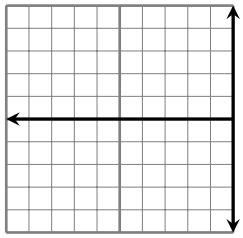
1. $y = x^2 - 4x + 5$



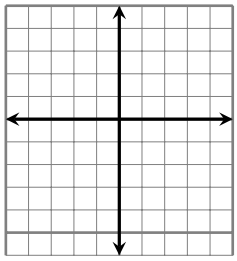
2. $y = x^2 + x + 1$



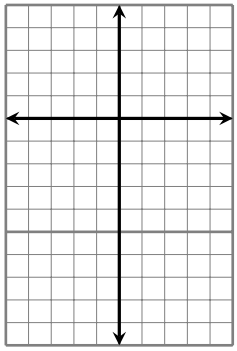
3. $y = 2x^2 + 16x + 28$



4. $y = -x^2 - 2x + 3$



5. $y = -3x^2 - 9x - 3$

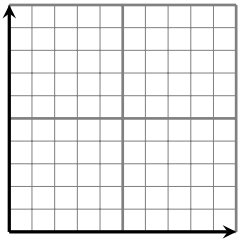


Circles

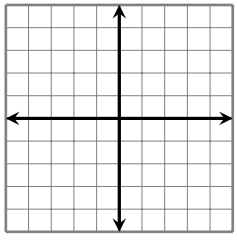
A ***circle*** is the set of points in a plane that are equidistant to a given point. The standard form of the equation of a circle of radius r centered at (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2.$$

6. Plot the circle: $(x - 4)^2 + (y - 5)^2 = 16$

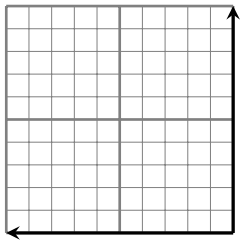


7. Plot the circle: $(x + 1)^2 + (y - 1)^2 = 9$

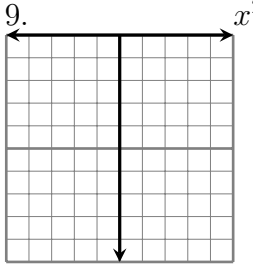


Graph each circle.

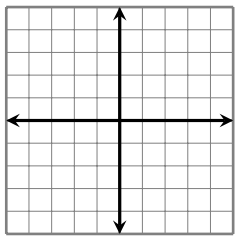
8. $x^2 + y^2 + 8x - 6y = -16$



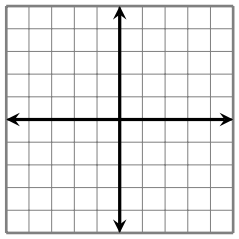
9. $x^2 + y^2 + 4x + 14y = -56$



10. $x^2 + y^2 = 0$



11. $x^2 + y^2 = -1$



Write the equation of each circle. Express the final equation in the form $x^2+y^2+Dx+Ey+F = 0$.

12. Center at $(2, -4)$ and $r = 9$

13. Center at $(1, 6)$ and $r = \sqrt{3}$

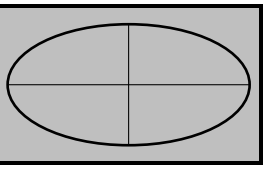
14. Center at $(-2, 3)$ and $r = 1$

15. Find the equation of the circle that passes through the origin and has its center at $(3, 4)$.

16. Find the equation of the circle that passes through the origin and has its center at $(-5, 12)$.

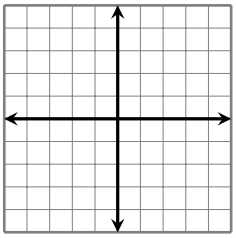
Ellipses

An ***ellipse*** that is centered at (h, k) is defined by an equation of the form $A(x-h)^2 + B(y-k)^2 = C$ $(A, B, C > 0)$. The longer line segment is called the ***major axis***, and the shorter is called the ***minor axis***.

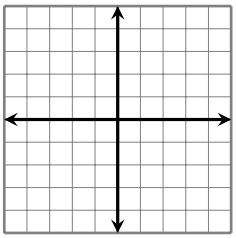


Graph each ellipse.

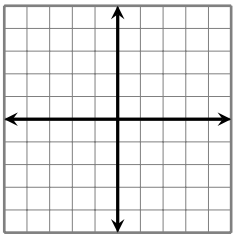
1. $9x^2 + 4y^2 = 36$



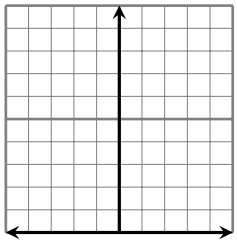
2. $x^2 + 4y^2 = 16$



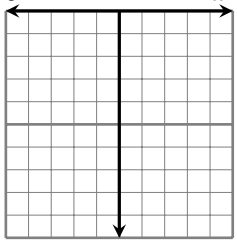
3. $9x^2 + 2y^2 = 36$



4. $9x^2 - 36x + 4y^2 - 24y + 36 = 0$



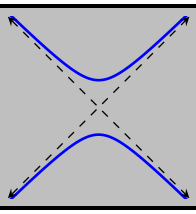
5. $4x^2 - 16x + y^2 + 8x + 16 = 0$



6. How are the graphs of $x^2 + 4y^2 = 36$ and $4x^2 + y^2 = 36$ related?

Ellipses

A ***hyperbola*** is the graph of the equation of the form $Ax^2 + By^2 = C$, where A and B are of unlike signs. Hyperbolas are characterized by two symmetric parts that are bounded by a pair of asymptotes. The asymptotes are given by the equation $Ax^2 + By^2 = 0$.



Find the equations of the asymptotes for the following hyperbolas.

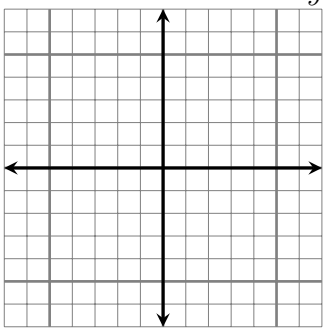
1. $x^2 - y^2 = 1$

2. $4x^2 - 9y^2 = 16$

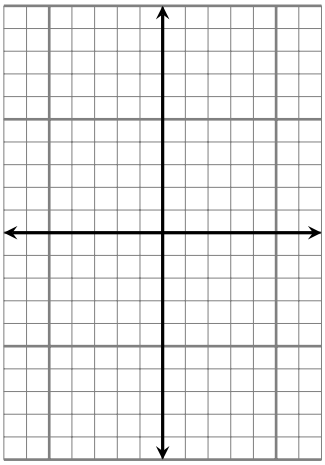
3. $y^2 - 8y - x^2 - 4x + 3 = 0$

Graph each hyperbola.

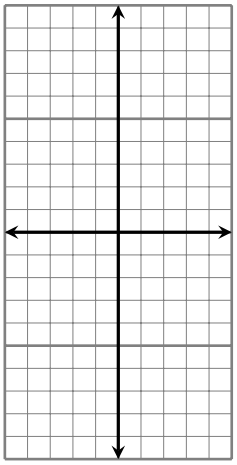
4. $y^2 - x^2 = 4$



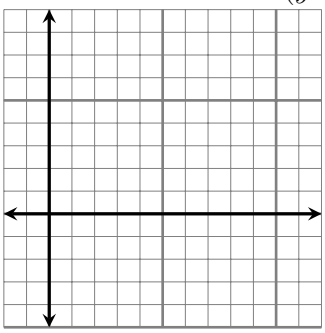
5. $4x^2 - y^2 = 16$



6. $4y^2 - 25x^2 = 64$

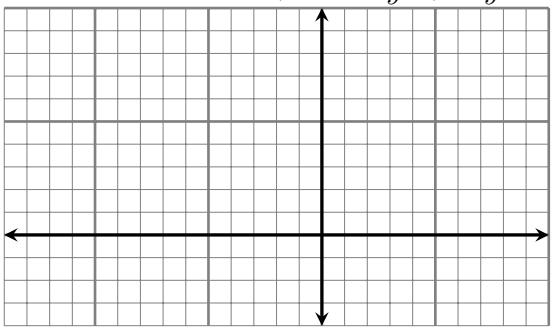


7. $(y-2)^2 - (x-5)^2 = 4$

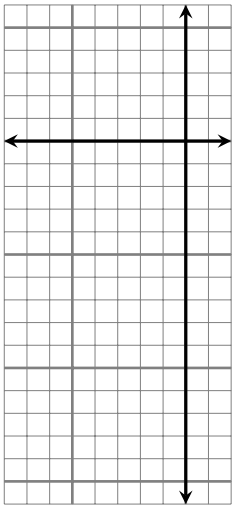


Graph each hyperbola.

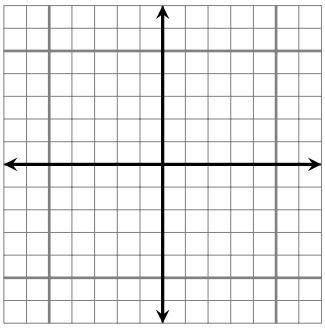
8. $x^2 + 4x - 9y^2 + 54y = 113$



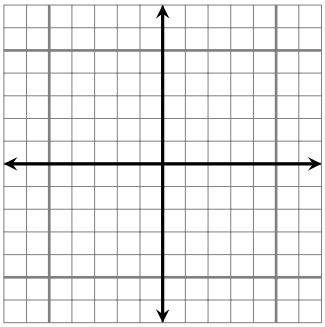
9. $y^2 + 6y - 4x^2 - 24x = 63$



10. $xy = 1$



11. $xy = -4$



State whether the graph of each equation is a circle, ellipse, parabola, hyperbola, or none of those options.

12. $x^2 + 2x + 3y = 16$

13. $y^2 + y + 3x^2 = 7$

14. $x^2 + 2xy + y^2 = 1$

15. $4x^2 + 4y^2 = 16$

16. $-5x^2 - 3y^2 = -8$

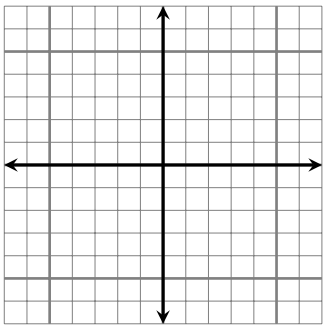
17. $-7x^2 + 47x + 2y^2 + y = 800$

Solutions of Linear Systems

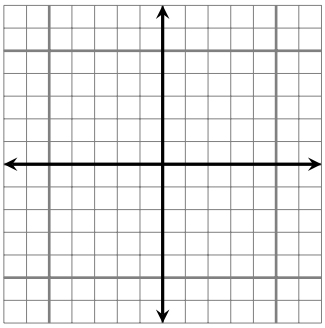
A ***linear equation*** in two variables has the form $Ax + By = C$. When we solve a system of equations, we find all ordered pairs that satisfy each of the equations in the system. This is equivalent to saying that the solution of a system of equations is the set of all points that are shared by the graphs of the equations.

Solve the systems of equations by graphing.

1. $2x + y = 4$
   $x - y = -1$



2. $y = 2x - 3$
   $y = 3x - 4$



The Substitution Method

1. Solve one equation for one variable in terms of the other variable.
2. Substitute in place of that variable in the other equation.
3. Solve that equation.
4. Plug that solution into the other equation to get the value of the other variable.

Solve each system of equations.

3. $x - y = -4$
   $3x + 2y = 8$

4. $9x - 2y = -38$
   $5x + y = 0$

5. $5x - y = 9$
   $10x - 2y = -6$

6. $-x + 4y = -22$
   $-4x + 7y = -41$

Solve each system of equations.

7. $\begin{aligned} 2x + 3y &= 7 \\ 3x + 2y &= 8 \end{aligned}$

8. $\begin{aligned} x - 1 &= y + 1 \\ -x &= 5x - 10y \end{aligned}$

9. $\begin{aligned} \frac{1}{2}x + \frac{1}{3}y &= 5 \\ \frac{2}{3}x - \frac{1}{2}y &= 1 \end{aligned}$

10. The perimeter of a rectangle is 32 inches. The length of the rectangle is 4 inches more than the width. find the dimensions of the rectangle.

11. Suppose that Gus invested \$8,000, part of it at 8% and the remainder at 9%. His yearly income from the two investments was \$690. How much did he invest at each rate?

12. Gallant has \$100 in his bank account, and each week he adds an additional \$30 to his account. Goofus has \$350 in his bank account. He spends \$20 from the account each week. How many weeks will it take for Gallant's savings to catch up with Goofus's savings?

The Elimination Method

1. Choose one variable to eliminate.
2. Multiply both equations by suitable numbers so that the coefficients of that variable are negatives of each other.
3. Add the two equations to get a new equation.
4. Solve the new equation.
5. Plug that solution into the other equation to get the value of the other variable.

Solve each system of equations.

1. $$\begin{aligned} x - y &= -4 \\ x + y &= 8 \end{aligned}$$

2. $$\begin{aligned} 3x + y &= 4 \\ 2x + 2y &= 4 \end{aligned}$$

3. $$\begin{aligned} 3x + 2y &= 22 \\ 2x + 3y &= 13 \end{aligned}$$

4. $$\begin{aligned} \tfrac{1}{2}x + \tfrac{1}{3}y &= 5 \\ 5x - 2y &= 18 \end{aligned}$$

5. $$\begin{aligned} 2x - 4y &= 10 \\ -4x + 8y &= -20 \end{aligned}$$

6. It costs \$80 to buy 2 football tickets and 1 basketball ticket. It costs \$130 to buy 3 football tickets and 2 basketball tickets. How much will it cost to buy 1 football ticket and 1 basketball ticket?

7. At a certain store Ding Dongs cost \$3 per box, and Twinkies cost \$4 per box. If 17 total boxes were sold for a total of \$55, then how many boxes of Twinkies were sold?

Solve each system using any method.

8. $\begin{aligned} 4x - 3y &= 1 \\ 2x - y &= -1 \end{aligned}$

9. $\begin{aligned} 8x + 7y &= 4 \\ 6x - 3y &= 3 \end{aligned}$

10. $\begin{aligned} y &= -8x - 54 \\ y &= 3x + 34 \end{aligned}$

11. $\begin{aligned} \frac{3x}{4} - \frac{2y}{3} &= 7 \\ \frac{7x}{5} + \frac{y}{4} &= 22 \end{aligned}$

12. A rectangular quidditch field is twice as long as it is wide, and its perimeter is 630 feet. What are the dimmensions of a quidditch field?

13. A pouch of coins contains nickels and dimes. The pouch contains 29 coins for a total of \$1.70. How many dimes are in the pouch?

14. The units digit of my age is one more than three times the tens digit. The sum of the two digits is 9. How old am I?