2305/301, 2308/301
2306/301, 2309/301
2307/301
MATHEMATICS
Oct./Nov. 2016
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN BUILDING
DIPLOMA IN QUANTITY SURVEYING
DIPLOMA IN CIVIL ENGINEERING
DIPLOMA IN HIGHWAY ENGINEERING
DIPLOMA IN ARCHITECTURE

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet
Mathematical tables/calculator
Drawing instruments

*Answer **FIVE** questions of the **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are shown.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Given that $Z_1 = 3 + 2j$; $Z_2 = -3 + 5j$ and $\frac{1}{Z_3} = \frac{1}{Z_1} + \frac{1}{Z_2}$; determine Z_3 in the form $a + bj$. (4 marks)

(b) Use Demoirve's theorem to find the following in terns of $\cos\theta$ and $\sin\theta$:

(i) Cos 3θ

(ii) Sin 3θ and hence

(iii) Show that $\tan 3\theta = \frac{3t - t^3}{1 - 3t^2}$ where $t = \tan\theta$.

(6 marks)

(c) Given that $Z^4 = -8 - 8\sqrt{3}j$, find Z in the form $a + bj$. (10 marks)

2. (a) Given that $\underset{\sim}{A} = p\underset{\sim}{i} - 6\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{B} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$, where p is a constant. Determine the value of p such that vectors $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are perpendicular to each other.

(4 marks)

(b) Determine the area of the triangle whose two sides are the vectors:

$\underset{\sim}{A} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k}$

$\underset{\sim}{B} = \underset{\sim}{i} + 5\underset{\sim}{j} - 2\underset{\sim}{k}$

$\sqrt{4^2+3^2-1^2} = P^2+6^2-3^2$

$Pi - 6j - 3k$ $\quad$ $P \; -6 \; -3$

$4i + 3j - k$ $\quad$ $4 \; 3 \; -1$

$5.099 = \sqrt{P^2+6+}$

(6 marks)

(c) Given that $6\sin\theta - 8\cos\theta = R\sin(\theta - \alpha)$ where $R > 0$ and $0^\circ \le \alpha \le 90^\circ$.

(i) Determine the values of R and α and hence

(ii) Solve the equation $6\sin\theta - 8\cos\theta = 5$ where $0^\circ \le \theta \le 360^\circ$.

(10 marks)

3. (a) Given that $V = x^3 \cos\left(\frac{y}{x}\right)$, show that

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = 3V$$

(5 marks)

$\frac{\partial v}{\partial r}$

(b) A right circular cylinder has radius, $r = 10$ cm and height, $h = 100$ cm. If the radius is increasing at the rate of 0.5 cm/s, determine the rate at which the volume is changing.

$V = \pi r^2 h$

(5 marks)

$\frac{\partial v}{\partial h} =$

(c) Given that $t = e^{2h} \ln(4h + 5p)$. Find the change in the value of t if h is increased from 0.4 cm to 0.44 cm and p is decreased from 0.3 cm to 0.28 cm.

(10 marks)

4. (a) (i) Given that $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$, evaluate

$\int_0^{\frac{\pi}{3}} \cos^4\theta \, d\theta$, giving your answer correct to two decimal places.

(ii) Evaluate $\int_2^3 \frac{5x^2+15x-2}{(x-1)(x+2)^2}\, dx$

(13 marks)

(b) (i) Determine the area of the region enclosed by $y = 6\sin 3x$, and x-axis between $x = 0$ and $x = \frac{\pi}{3}$.

(ii) Find the volume generated when the area in b(i) is rotated about the x-axis through 360°.

(7 marks)

5. (a) Solve the differential equation $2xy\frac{dy}{dx} = x^2 + y^2$, given that when $x = 3$, $y = 2\sqrt{2}$.

(8 marks)

(b) Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4e^{3x}$, $m^2+3m+2=0$

given that when $x = 0, y = 2$ and $\frac{dy}{dx} = 0$.

(12 marks)

6. (a) Given that

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -1 & 2 \\ 1 & -1 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 5 & 1 \\ 1 & -2 & -3 \\ 3 & -1 & -2 \end{bmatrix}$$

Show that $(AB)^T = B^T A^T$

(7 marks)

(b) Use inverse matrix method to solve the simultaneous equations

$2x - 3y + 2z = 9$.

$3x + 2y - z = 4$.

$x - 4y + 2z = 6$.

(13 marks)

7. (a) A random sample of size 100 from an infinite population has a mean of 80 and a standard deviation of 15.

Determine the probability that the sample mean lies between 79 and 82.

(6 marks)

(b) A continuous Random variable x has probability density function defined by

$$f(x)=\begin{cases} Kx^3; & 0<x<1 \\ K; & 1<x2 \\ O; & elsewhere \end{cases}$$

Determine:

(i) the value of the constant K.

(ii) the mean and standard deviation of x.

(iii) $P(x<\frac{1}{2})$

(14 marks)

8. (a) Two major brands of light bulbs A and B are used in an apartment. A sample of 50 bulbs of brand A revealed a mean life-time of 6.94 months and a standard deviation of 0.82 months. A sample of 60 bulbs from brand B revealed a mean life time of 7.34 months and a standard deviation of 1.53 months.
Test at 5% level of significance the claim that there is no difference in quality between the two brands of bulbs.

(7 marks)

(b) A experiment was carried out on small cantilevered steel beam, various masses were placed on the end of the beam and corresponding deflections measured as shown in **table 1**.

Table 1

Mass x (grammes)	Deflection y (mm)
0	0
50.15	0.6
99.90	1.8
150.05	3.0
200.05	3.6
250.20	4.8
299.95	6.0
350.05	6.2
401.00	7.5

(i) Find the least squares line of Regression of y and x.

(ii) Predict the deflection when the mass is 220g.

(13 marks)

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