2305/301 2308/301
2306/301 2309/301
2307/301
MATHEMATICS
Oct./Nov. 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN BUILDING
DIPLOMA IN QUANTITY SURVEYING
DIPLOMA IN CIVIL ENGINEERING
DIPLOMA IN HIGHWAY ENGINEERING
DIPLOMA IN ARCHITECTURE

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Mathematical table / calculator;*
- *Drawing instruments;*
- *Answer booklet.*

*This paper consists of **EIGHT** questions.*
*Answer **FIVE** questions in the answer booklet provided.*
***All** questions carry equal marks.*
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Prove the identities:

(i) $\frac{1-\sin\theta}{1+\sin\theta}=\frac{1}{(\sec\theta+\tan\theta)^2}$

(ii) $\tan 3A=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$

(9 marks)

(b) Prove that $\sin\theta+\sin(\theta+120^\circ)+\sin(\theta+240^\circ)=0$

(4 marks)

(c) (i) Express $12\cos\theta+5\sin\theta$ in the form $R\cos(\theta-\alpha)$, where $R>0$ and $0^\circ\leq\alpha\leq 90^\circ$.

(ii) Hence solve the equation $12\cos\theta+5\sin\theta=6.5$, for values of θ between 0° and 360° inclusive.

(7 marks)

2. (a) Write down the middle term in the binomial expansion of $(x+2y)^{12}$, and determine its value when $x=\frac{1}{2}$ and $y=\frac{2}{3}$.

(7 marks)

(b) Determine the first four terms in the binomial expansion of $\left(1-\frac{2}{3}x\right)^{-\frac{1}{3}}$, and state the values of which the expansion is valid.

(4 marks)

(c) (i) Use the binomial theorem to show that, for very small value of x,

$\sqrt{\left(\frac{1+2x}{1-2x}\right)}=1+2x+2x^2+4x^3$

approximately.

(ii) By setting $x=\frac{1}{100}$ in (i) above, determine the approximate value of $\sqrt{51}$, correct to four decimal places.

(9 marks)

3. (a) Given the matrices $A=\begin{bmatrix}-1 & 1 & -1\\ 1 & 2 & -1\\ 1 & 0 & 2\end{bmatrix}$,

and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & 1\\ 0 & 1 & -1\end{bmatrix}$, determine $(AB)^{-1}$.

(10 marks)

(b) Three forces F_1, F_2 and F_3 in newtons, necessary for the stability of a structure satisfy the simultaneous equations

$$F_1 - F_2 + F_3 = 5$$
$$F_1 - 2F_2 + F_3 = 2$$
$$F_1 + F_2 - F_3 = -1$$

Use Cramer's rule to solve the equations.

(10 marks)

4. (a) Show that the general solution of the differential equation $(x^2 + y^2)\,dx - xy\,dy = 0$ may be expressed in the form $x = cy$, where c is an arbitrary constant.

(7 marks)

(b) Solve the differential equation

$$6\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = e^{-2x}\text{, that when } x = 0,\ y = 0 \text{ and } \frac{dy}{dx} = -1$$

(13 marks)

5. (a) Find $\frac{dy}{dx}$ from first principles, given that $y = \frac{3}{x^2}$.

(6 marks)

(b) Use implicit differentiation to determine the equation of the tangent to the curve $x^2 - 2y^2 + 3xy - 2x + 6y = 4$, at the point (0,2).

(6 marks)

(c) A curve has the parametric equations $x = \theta - \cos\theta$, $y = \sin\theta$. Determine the radius of curvature at $\theta = \pi$.

(8 marks)

6. (a) (i) Evaluate the integral:

$$\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx$$

(ii) Show that $\displaystyle\int_0^1 \frac{dx}{(x+1)(x^2+x+1)} = \ln\left(\frac{2}{\sqrt{3}}\right) + \frac{\pi}{6\sqrt{3}}$

(13 marks)

(b) Use integration to determine the length of the curve $y = \frac{1}{3}x^{\frac{3}{2}}$ between the points x = 1 and x = 2.
Correct two decimal places.

(7 marks)

7. (a) Given the complex numbers $z_1 = 2 - j$, $z_2 = 1 - 2j$ and $z_3 = 1 + j$, express

$$z = z_1 + \frac{z_1 + z_3}{z_1 z_3}$$ in polar form.

(8 marks)

(b) Find all the roots of the equation

$$z^3 + 3z^2 + 4z + 2 = 0.$$

(6 marks)

(c) Use De Moivre's theorem to prove that

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

(6 marks)

8. (a) **Table 1** shows data obtained from an experiment to determine the relationship between the force (x) and extension (y) on a cable.

Table 1

Force N(x)	10	20	30	40	50	60	70
Extension mm (y)	0.12	0.30	0.51	0.75	1.10	1.35	1.60

Calculate Karl Pearson's correlation coefficient.

(10 marks)

(b) A continuous random variable X has a probability density function defined by

$$f(x) = \begin{cases} k, & 0 \le x \le 1 \\ k(4x - 2), & 1 \le x \le 2 \\ 0, & elsewhere \end{cases}$$

Determine the:

(i) value of the constant k,

(ii) mean,

(iii) $P(x \ge 1.5)$

(10 marks)

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