2305/301 2308/301
2306/301 2309/301
2307/301
MATHEMATICS
Oct./Nov. 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN BUILDING
DIPLOMA IN QUANTITY SURVEYING
DIPLOMA IN CIVIL ENGINEERING
DIPLOMA IN HIGHWAY ENGINEERING
DIPLOMA IN ARCHITECTURE

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Scientific calculator.
*Answer **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all pages are printed as indicated and that no questions are missing.

1. (a) Determine the binomial expansion of $(4+3x)^{-\frac{1}{2}}$ as far as the term in x^3 and state the values of x for which the expansion is valid. (6 marks)

(b) (i) Use the binomial theorem to expand $\left(\dfrac{1+\frac{1}{5}x}{1-\frac{1}{5}x}\right)^{\frac{1}{2}}$ as far as the third term.

(ii) By setting $x=\frac{1}{13}$ in the result in (i), determine the approximate value of $\sqrt{66}$, correct to four decimal places. (9 marks)

(c) The sum of the first six terms of an arithmetical progression is 21, and the seventh term is three times the sum of the third and fourth. Determine the first term and the common difference. (5 marks)

2. (a) Given the matrices $A=\begin{bmatrix}1 & 2\\ -1 & 3\\ 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}-1 & 1 & 2\\ 1 & -1 & -3\end{bmatrix}$ and $C=\begin{bmatrix}1 & -1 & 2\\ -1 & 1 & 1\\ 0 & 1 & -2\end{bmatrix}$

Determine:

(i) $D=AB-C$;

(ii) adjoint D. (9 marks)

(b) A structure is subjected to three forces F1, F2 and F3 in newtons, satisfying the simultaneous equations:

$$F_1-F_2+2F_3=3$$
$$-2F_1+F_2+F_3=-2$$
$$F_1+F_2-F_3=2$$

Use the inverse matrix method to solve the equations. (11 marks)

3. (a) Solve the differential equation:

$$x\frac{dy}{dx}+(1+x)y=x,\text{ given that when } x=1,\ y=0.$$ (9 marks)

(b) A particle moves in such a way that its displacement from a fixed position satisfies the differential equation:

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=e^{t}$$

Use the method of undetermined coefficients to find an expression for $x(t)$. (11 marks)

4. (a) Given the complex numbers $z_1 = 2 - 3j$, $z_2 = 2 + j$ and $z_3 = -1 + j$

Determine;

(i) $3z_1 - 2z_3$;

(ii) $z = \dfrac{2z_1z_2}{z_1 3z_3}$. (8 marks)

(b) The vectors $V_1 = -4i + 2j + pk$, $V_2 = 4i + j + 5k$ and $V_3 = 4i - 2j + 8k$ are coplanar. Determine the value of the constant p. (6 marks)

(c) Determine the angle between the $A = i + 2j + 2k$ and $B = 2i + 2j - k$. (6 marks)

5. (a) Find $\dfrac{dy}{dx}$ from first principles, given $y = \sin 2x$. (5 marks)

(b) Given $u = \ln(x^2 + y^2)^{\frac{1}{2}}$, show that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$. (6 marks)

(c) Determine the maximum and minimum values $f(x) = x^3 + 6x^2 + 9x - 2$. (9 marks)

6. (a) Given that $\sin A = \dfrac{3}{5}$ and $\cos B = \dfrac{5}{13}$ and that A is obtuse and B is acute, determine

(i) $\sin(A + B)$;

(ii) $\cos(A - B)$. (6 marks)

(b) (i) Prove that $\tan(A + B + C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan B \tan C - \tan C \tan A - \tan A \tan B}$

(ii) Hence, deduce that, if A, B and C are angles of a triangle, then $\tan A + \tan B + \tan C = \tan A \tan B \tan C$. (7 marks)

(c) (i) Express $\cos\theta - 7\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$.

(ii) Hence solve the equation

$\cos\theta - 7\sin\theta = 3$, for the values of θ between 0° and 360° inclusive. (7 marks)

7. (a) Evaluate the integrals:

(i) $\displaystyle\int_0^1 \frac{2x + 1}{x^2 + 2x + 2}\,dx$;

(ii) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1 + \cos^2 x}\,dx$. (12 marks)

(b) (i) Sketch the region bounded by the curve $y = 3 - x^2$ and the line $y = 2x$.

(ii) Use integration to determine the area of the region in (i). (8 marks)

8\. (a) **Table 1** shows the surrounding temperature and the time it takes for a concrete mixture to dry.

Temperature 0°C	Time (Days)
17	23
18	22
20	20
24	18
16	26
27	17
19	20
30	16
32	14
28	16

(i) Calculate the Karl Pearson's product moment correlation coefficient.

(ii) Hence comment on the relationship between the temperature and the time it takes for the mixture to dry. (9 marks)

(b) A continuous random variable t has a probability density function defined by

$$f(t) = \begin{cases} c(1-t)^2, & 1 < t < 3 \\ 0, & elsewhere \end{cases}$$

Determine the:

(i) value of the constant c;

(ii) mean;

(iii) $P(1.2 \leq t \leq 2.2)$. (11 marks)

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