2405/301
MATHEMATICS
Oct./Nov. 2009
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN APPLIED STATISTICS

MATHEMATICS

3 hours

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) If $z = x \sin(x+y)$, find $\frac{dz}{dt}$, when $x = \frac{\pi}{2}$, $y = \frac{\pi}{2}$, $\frac{dx}{dt} = \frac{1}{2}$ and $\frac{dy}{dt} = \frac{1}{3}$

(5 marks)

(b) Determine and classify the turning points of the function:

$$z = 6x^2 + 6xy + 9y - 18x - y^3$$

(11 marks)

(c) If $T = 2\pi\sqrt{\frac{m}{g}}$, show that $\frac{\delta T}{T} = \frac{1}{2}\left(\frac{\delta m}{m} - \frac{\delta g}{g}\right)$ using partial derivatives. Hence find the percentage change in T when m increases by 2% and g decreases by 3%.

(4 marks)

2. Solve the following differential equations:

(a) $(3x - 2y)\frac{dy}{dx} = (2x + y)$ (10 marks)

(b) $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 2e^{-2t}$ (10 marks)

3. (a) If A = $\begin{pmatrix} 2 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{pmatrix}$

(i) find AA^T;

(ii) deduce A^{-1} from the product AA^T. (5 marks)

(b) Given the matrices

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 1 & 4 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 3 & 1 \\ 1 & -2 & 3 \\ 1 & 4 & 1 \end{pmatrix}$$

find $(AB)^{-1}$ using cofactors. (8 marks)

(c) Use Cramers rule to solve the simultaneous equations:

$$3x + 4y + z = 4$$
$$x - 2y - 5z = -18$$
$$x + y + 2z = 6$$

(7 marks)

4. (a) Given that

$\sin^{-1}(x) = \int g(x)\,dx$, show that $\sin^{-1}(x) = x + \frac{x^3}{3} + \frac{3x^5}{40} + \frac{5x^7}{12} + \text{---}$

(4 marks)

(b) Obtain the Maclaurin's series for $f(x) = 1 - e^{-2x}$ up to the term in x^4

Hence evaluate $\int_0^1 x^3(1-e^{-2x})\,dx$ correct to five decimal places.

(11 marks)

(c) Use the Taylor's theorem to find an approximate value for sin 30.05

(5 marks)

5. (a) Find the cube roots of $z = 1 - j$ in the form $a + bj$. (7 marks)

(b) Given that $z = 1 - 4j$ is a root of the equation $z^4 - Az^3 + Bz^2 - 122z + 170 = 0$, find:

(i) the values of A and B;

(ii) the other roots. (13 marks)

6. (a) Using Newton-Raphson method with four iterations find the root of $x^3 - 3x^2 + 4 = 0$ near $x_0 = 1.5$. Give the answer correct to 3 decimal places.

(6 marks)

(b) Find and correct the wrongly recorded value in the following table.

x	0	0.1	0.2	0.3	0.4	0.5
f(x)	-1.000	-0.659	-0.232	0.287	0.940	1.625
x	0.6	0.7	0.8	0.9	1.0	
f(x)	2.456	3.403	4.472	5.669	7.00	

(14 marks)

7. (a) Evaluate:

(i) $$\int_0^{\frac{\pi}{2}} \int_0^{4\cos\theta} \sin\theta\, e^{2x}\, dx\, d\theta$$

(ii) $\int_0^1 \int_0^{1-x} \int_0^{2-x} (1+x)\, yz\, dz\, dy\, dx$.

(13 marks)

(b) Find the volume of the region bounded by the cylinder $x^2 + y^2 = 9$ and the planes $y + z = 9$ and $z = 0$. (7 marks)

8. (a) (i) Given that $\underset{\sim}{a} = 3i + 5j$ and $\underset{\sim}{b} = i + 2j$, find $\underset{\sim}{a}.\underset{\sim}{b}$ and hence calculate the angle between the two vectors.

(ii) Given $\underset{\sim}{a} = 3i + j + 5k$, $\underset{\sim}{b} = 7i - j + 2k$ find $\underset{\sim}{a} \times \underset{\sim}{b}$. (8 marks)

(b) If x is so small that x^3 and higher power of x could be neglected, determine $\sqrt{\left(\frac{1+2x}{1-x}\right)}$ and evaluate $\sqrt{2}$ by substituting $x = \frac{1}{4}$. (8 marks)

(c) Use the binomial theorem to evaluate $\frac{1}{2\sqrt{2}}$ correct to three decimal places.

(4 marks)