Pdf SMA 104 Lecture 11(Kinematics)

Project Management (University of Nairobi)

KINEMATICS

VELOCITY AND ACCELERATION

The velocity *(v)* is instantaneous rate of change of position. The velocity of a moving particle can be positive or a negative, depending on whether the particle is moving in the positive or negative direction along a line of motion.

Suppose a particle moves along a horizontal straight line, with its location at time t given by its position function $s = f(t)$. Think of the time interval from t to $t+\Delta t$. The particle moves from position $f(t)$ to position $f(t+\Delta t)$ during this interval. Then v is given by

$$v = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t} = \frac{ds}{dt} = f'(t)$$

Example: An object moving in a straight line has its displacement s meters from an origin 0 at time t seconds given by $s = t(t-3)^2$.

Determine a)The time when the object is at the origin

b)The time when the object is at rest

c)The distance moved between $t=0$ and $t=2$. Use $s = \sqrt{1+\left(\frac{ds}{dt}\right)^2}$.

Solution:

a) The object will be at the origin when $s=0$

$$0 = t(t-3)^2;\ 0 = t(t^2-6t+9);\quad t^3-6t^2+9t=0;\quad t(t^2-6t+9)=0;$$

$$t=0 \quad \text{or} \quad t^2-6t+9=0;\quad (t-3)^2=0 \qquad t=3\text{sec.}$$

b) $v = \dfrac{ds}{dt} = (t-3)^2 + 2(1)(t-3)'t$

$$v = \frac{ds}{dt} = t^2-6t+9+2t^2-6t = 3t^2-12t+9;\quad v=(t-3)(3t-3)$$

For max or min $v=0$; $(t-3)(3t-3)=0$; $t=3$ or $t=1$

The object is thus instantaneously at rest at $t=1$ and $t=3$ seconds.

(c)By second derivative

$$\frac{d^2v}{dt^2} = 6t-12;\ \frac{d^2v}{dt^2}\Big|t=3 \qquad = 18-12=6>0 \text{ minimum point.}$$

$$\frac{d^2v}{dt^2}\Big|t=1 \qquad = 6-12<0 \qquad \text{maximum point}$$

When $t=3$, $s=0$ when $t=0,\ s=0$

When $t=1, s=4$ when $s=0, t=3$

When $t=1$, $s=4$(distance)

Between $t=0,$ and $t=1$, the velocity is positive and the object moves from position $s=0$ to $s=1(1-3)^2=4$.

Between $t=1$ and $t=3$, the velocity is negative and the object moves from position $s=4$ to position $s=0$.

Therefore the distance moved by the object between $t=0$ and $t=2$ will be given by 4(the positive difference between values of s at time $t=1, t=2$ respectively).

when $t=1, s=4$
when $t=2, s=2$ $\quad \therefore$ total distance is (4+2)=6metres.

Acceleration *(a)* at a time t is given by $a=\frac{dv}{dt}$.

Hence to determine the acceleration at time t differentiate v with respect to t.

Examples:

1. A particle is moving in a straight line and has its displacement s metres from the origin after t seconds given by $s=e^{-\sqrt{3}t}\sin t$. Determine its displacement, velocity and acceleration when $t=\frac{\pi}{2}$ and also the smallest positive value t for which the particle is at rest (i.e. $v=0$).

Solution:

$$s=e^{-\sqrt{3}t}\sin t; \quad v=\frac{ds}{dt}=\cos te^{-\sqrt{3}t}+\left(-\sqrt{3}\right)e^{-\sqrt{3}t}\sin t$$

$$a=\frac{dv}{dt}=-\sqrt{3}e^{-\sqrt{3}t}\cos t+(-\sin t)e^{-\sqrt{3}t}+\left(-\sqrt{3}\right)\left(-\sqrt{3}\right)e^{-\sqrt{3}t}\sin t+\cos t\left(-\sqrt{3}\right)e^{-\sqrt{3}t}$$

$$=-\sqrt{3}e^{-\sqrt{3}t}\cos t-\sin te^{-\sqrt{3}t}+3e^{-\sqrt{3}t}\sin t+\cos t\left(-\sqrt{3}\right)e^{-\sqrt{3}t}$$

$$a=-2\sqrt{3}e^{-\sqrt{3}t}\cos t+2e^{-\sqrt{3}t}\sin t$$

$$\therefore \text{ at } t=\frac{\pi}{2}, \quad v=\left.\frac{ds}{dt}\right|t=\frac{\pi}{2}=-\sqrt{3}e^{-\sqrt{3}\frac{\pi}{2}}\sin\frac{\pi}{2}+e^{-\sqrt{3}\frac{\pi}{2}}\cos\frac{\pi}{2} \quad =-\sqrt{3}e^{-\sqrt{3}\frac{\pi}{2}}$$

$$a=\left.\frac{dv}{dt}\right|t=\frac{\pi}{2} \quad =2e^{-\sqrt{3}\frac{\pi}{2}}-2\sqrt{3}e^{-\sqrt{3}\frac{\pi}{2}}\cdot 0=2e^{-\sqrt{3}\frac{\pi}{2}}$$

The displacement at $t=\frac{\pi}{2}$ is given by $s=e^{-\sqrt{3}t}\sin t$; $s=e^{-\sqrt{3}\frac{\pi}{2}}\sin\frac{\pi}{2}$; $s=e^{-\sqrt{3}\frac{\pi}{2}}$ metres.

When the particle at rest $v=0$; $\therefore v=-\sqrt{3}e^{-\sqrt{3}t}\sin t+e^{-\sqrt{3}t}\cos t=0$

$v=e^{-\sqrt{3}t}\left(-\sqrt{3}\sin t+\cos t\right)=0$; $e^{-\sqrt{3}t}>0$ for all t

$$-\sqrt{3}\sin t+\cos t=0 \quad ; \; -\sqrt{3}\sin t=-\cos t$$

$$\frac{\sin t}{\cos t}=\frac{1}{\sqrt{3}} \quad ;\tan t=\frac{1}{\sqrt{3}}; \quad t=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=30^0, 210^0; \quad \therefore \text{ the smallest is } t=30^0.$$

2. A distance time graph is represented by the equation $s=2t^3-2t^2-3t$.
   Evaluate (a)The velocity at time t
   (b)The acceleration at time t

(c)Show that the minimum distance over attained occurs went $t=\frac{2+\sqrt{22}}{6}$

Solution:

(a) $v=\frac{ds}{dt}=6t^2-4t-3$

(b) $a=\frac{dv}{dt}=12t-4$

(c) for minimum and maximum distance *v=0*

$v=6t^2-4t-3=0$; $t=\frac{4\pm\sqrt{16+72}}{12}$ $=\frac{4\pm\sqrt{88}}{12}$ $=\frac{4\pm2\sqrt{22}}{12}$; $t=\frac{2\pm\sqrt{22}}{6}$

$\therefore t=\frac{2+\sqrt{22}}{r}$ or $t=\frac{2-\sqrt{22}}{6}$

Using the second derivation test

$a=\frac{dv}{dt}=12t-4$; $\left.\frac{dv}{dt}\right|t=\frac{2-\sqrt{22}}{6}$ $=12\left(\frac{2-\sqrt{22}}{6}\right)-4=-9\cdot4<0$ maximum

$\left.\frac{dv}{dt}\right|t=\frac{2+\sqrt{22}}{6}$ $=12\left(\frac{2+\sqrt{22}}{6}\right)-4=9\cdot4>0$ minimum

Exercise

1. A particle moves along a straight line in such a way that after *t* seconds, its velocity is $v$ms^{-1}, where $v=t^2-t+2$. Find the acceleration the particle(a) after 2 seconds(b) after $\frac{1}{2}$ seconds.

2.The distance *s* metres that a particle has gone in *t* seconds is given by $s=5t+15t^2-t^3$.Find the velocity and
acceleration after (a)3seconds (b)6seconds .When is the acceleration zero?

3. After *t* seconds a particle has gone *s* metres where $s=t^3-6t^2+9t+5$.
(a) After how many seconds is its velocity zero?
(b) When is its acceleration zero?
(c)Find its velocity and acceleration (i)initially (ii)after 4seconds

In summary, the motion of a particle *p* along a straight line is completely described by the equations $s=f(t)$, where *t >0* is the time and *s* is the distance of *p* from a fixed point 0 in its path.

The velocity of *p* at time *t* is $v=\frac{ds}{dt}$.

If *v=0*, *p* is instantaneously at rest. Acceleration of *p* at time *t* is $a=\frac{dv}{dt}=\frac{d^2s}{dt^2}$