



SMA104 Past papers Courtesy of Jeff

ECONOMICS & Stat (Kenyatta University)



KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2017/2018

DIGITAL SCHOOL OF VIRTUAL AND OPEN LEARNING

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF  
SCIENCE

SMA 104: CALCULUS I

DATE: Saturday 16<sup>th</sup> December 2017

TIME: 2.00p.m -4.00p.m

**Instructions: Answer Question One and any other two questions**

**Question One [30 marks]**

- (a) Differentiate  $y = 3x^2 + 2x - 10$  using first principles. [5 marks]
- (b) Differentiate  $f(x) = xe^x \sin x$  [5 marks]
- (c) If  $x^2y^2 - \frac{y^3}{x} = 66$  find  $\frac{dy}{dx}$  at  $(-4, 2)$ . [5 marks]
- (d) Find  $\frac{d^2y}{dx^2}$  in terms of  $t$ , given that  $x = b\sin^3 t, y = b\cos^3 t$ . [5 marks]
- (e) Examine  $f(x) = x^3 + 2x^2 - 4x - 8$  for maxima and minima using the second derivative. [5 marks]
- (f) Use differentials to approximate  $\sqrt[3]{1020}$ . [5 marks]

**Question Two [20 marks]**

- (a) A reservoir has the shape of an inverted cone whose cross-section is an equilateral triangle. If water is pumped out of the reservoir at the rate of  $2m^3/sec$ , at what rate is the height of water changing when  $h$  is 40m? [5 marks]
- (b) Evaluate  $\lim_{x \rightarrow \infty} \frac{4x^2 - x}{2x^3 - 5}$  [5 marks]

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(c) If  $4x^2 - 2y^2 = 9$  find  $\frac{d^2y}{dx^2}$  [5 marks]

(d) Find the gradient of the curve  $x = \frac{t}{1+t}$ ,  $y = \frac{t^3}{1+t}$  at the point  $\left(\frac{1}{2}, \frac{1}{2}\right)$  [5 marks]

**Question Three [20 marks]**

(a) Use  $y = \ln x$  to approximate the value of  $\ln 1.1$ . [5 marks]

(b) Find  $\frac{dy}{dx}$  where  $e^y = x^2 - 3$  [5 marks]

(c) Find the gradient of the curve  $x = \frac{t}{1+t}$ ,  $y = \frac{t^3}{1+t}$  at the point  $\left(\frac{1}{2}, \frac{1}{2}\right)$  [5 marks]

(d) Use  $y = \ln x$  to approximate the value of  $\ln 1.1$ . [5 marks]

**Question Four [20 marks]**

(a) Given that  $f(0) = 8$ ,  $f'(0) = 3$ ,  $g(0) = 5$ ,  $g'(0) = 1$ , find  $F'(0)$  where  $F(x) = \frac{f(x)}{g(x)} + 4x^2 + 4x$  [7 marks]

(b) An object moves along a coordinate line. Its position at each time  $t$  is given by  $x(t) = \frac{2t}{t+3}$ . Find the velocity and acceleration at time  $t = 3$ . [7 marks]

(c) If the radius of a sphere is increasing at the constant rate of 5 millimeters per second, how fast is the volume changing when the surface area is 15 square millimeters? [6 marks]

**Question Five [20 marks]**

(a) Define the continuity of a function  $f(x)$  at a point  $x = a$  [3 marks]

(b) Differentiate the function  $f(x) = \sin(x)$  from first principles. [7 marks]

(c)  $y = x^{\sin x} + a^x$ , where  $a$  is a constant [5 marks]

(d)  $y = \ln(\ln x)$ . [5 marks]

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**KENYATTA UNIVERSITY**  
**UNIVERSITY EXAMINATIONS 2014/2015**  
**DIGITAL SCHOOL OF VIRTUAL AND OPEN LEARNING**  
**SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF**  
**SCIENCE**

**SMA 104: CALCULUS I**

**DATE:** Tuesday 28<sup>th</sup> April, 2015

**TIME:** 2.00 p.m – 4.00 p.m

**INSTRUCTIONS:**

*Attempt question ONE and any other TWO questions*

**QUESTION ONE (30 MARKS)**

- (a) Using the precise definition of limits show that

$$\lim_{n \rightarrow \infty} \left\{ \frac{(-1)^n}{n} \right\} \rightarrow 0 \text{ where } n \in \mathbb{N}. \quad \text{Hence evaluate } N(\epsilon) \text{ when}$$

(i)  $\epsilon = 0.02$

(ii)  $\epsilon = 0.006$  (4 marks)

- (b) Explaining every step evaluate the following limits

(i)  $\lim_{n \rightarrow \infty} \frac{3n^2 - 4n + 9}{2n^2 + 5}$  (2 marks)

(ii)  $\lim_{x \rightarrow 0} \frac{\sqrt[n]{1+ax} - \sqrt[n]{1+bx}}{x}$  (2 marks)

(iii)  $\lim_{x \rightarrow \infty} \frac{(x-3)^{40} (5x+6)^{10}}{(3x^2-6)^{25}}$  (2 marks)

(iii)  $\lim_{x \rightarrow 0} \frac{x^x \sin 2x}{3x}$  (2 marks)

- (c) From the first principles find  $\frac{dy}{dx}$  of the following functions; (3 marks)
- (i)  $f(x) = \cos x$  (3 marks)
- (ii)  $f(x) = \sqrt{x+2}$
- (d) Let  $\lim_{n \rightarrow \infty} X_n \rightarrow A$  and  $\lim_{n \rightarrow \infty} y_n \rightarrow B$  Show that (4 marks)
- (i)  $\lim_{n \rightarrow \infty} x_n + y_n = A + B$  (4 marks)
- (ii)  $\lim_{n \rightarrow \infty} x_n y_n = A \cdot B$  (4 marks)
- (e) Deduce the quotient rule of differentiation (4 marks)

## QUESTION TWO (20 MARKS)

- (a) Given  $y = \frac{\cos x}{x}$ , hence or otherwise prove that  $x \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + xy = 0$  (4 marks)
- b) For what value of the constant B is the following function continuous for all  $x \in \mathbb{R}$

$$\text{i) } F(x) = \begin{cases} x^2 - 3x + 4 & \text{if } x > 1 \\ Bx + 5 & \text{if } x < 1 \end{cases}$$

$$\text{ii) } F(x) = \begin{cases} 5x + B & \text{if } x > 3 \\ (x+B)^2 & \text{if } x < 3 \end{cases}$$

(6marks)

- (c) let  $f(x) = \sqrt{x}$ ,  $g(x) = 4x - 8$  and  $h(x) = 2x$  find  $f \circ g \circ h$  and hence find  $(f \circ g \circ h)^{-1}$  (2marks)

- (d) Determine whether  $y = Ae^{ax} + Be^{-ax}$  is satisfied by  $y'' - a^2 y = 0$  (3 marks)

- (e) Find the equation of the curve given the gradient is  $4x - 2$  and  $x$  - axis is the tangent.

(5 marks)

### QUESTION THREE (20 MARKS)

- (a) A particle moves along a straight line in such a way that its distance from a fixed point on the line after  $t$  seconds is  $S$  meters, where  $S = \frac{1}{6}t^3$ . Find;
- (i) Its velocity after 2 seconds and 3 seconds
  - (ii) Its acceleration after 1 seconds and 4 seconds
- (5 marks)**
- (b) A ball was thrown upwards with a velocity of 40 m/s. Find
- (i) The acceleration, velocity and distance statements.
  - (ii) The maximum height the ball can attain (strictly use calculus techniques)
- (6 marks)**
- (c) The volume of a cylindrical tank is  $32\pi \text{ m}^3$ . Find the radius of the tank if the area have to be least.
- (5 marks)**
- (d) A 525 meter wire mesh was provided to fence a rectangular (like) plot. Find the maximum area it can enclose by mesh without any loss.
- (4 marks)**

### QUESTION FOUR (20 MARKS)

- (a) Use the first principle to find  $\frac{dy}{dx}$  of the following functions
- (i)  $y = \ln x$  **(4 marks)**
  - (ii)  $y = \cos x$  **(4 marks)**
- (b) Given  $y = 2x^3 - 15x^2 + 24x + 19$  find the stationary points. **(4 marks)**
- (c) Differentiate the following functions.
- (i)  $y = \sin^3 2x$  **(2 marks)**
  - (ii)  $y = \frac{\tan^2 e^x \{\cos x\}}{x^2}$  **(2 marks)**
- (d) Evaluate
- (i)  $\lim_{x \rightarrow \infty} \left\{ \frac{(3x+1)}{3x-2} \right\}^{2x}$  **(2 marks)**
  - (ii)  $\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2}$  **(2 marks)**

**QUESTION FIVE (20 MARKS)**

- (a) Find  $y'$  and  $y''$  given  $y = \frac{e^{ax} + e^{-ax}}{e^{ax} - e^{-ax}}$  (6 marks)
- (b) Find  $y''$  given  $y = e^{-2x} \sin 4x$  (5 marks)
- (c) Find  $y'$  and  $y''$  given  $x^4 + xy^3 + y^3 = 32$  at the point  $(1, 1)$  (5 marks)
- (d) Show that  $\lim_{x \rightarrow 2} x^2 + 2x + 8 = 16$  (4 marks)



# KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2017/2018

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

## SMA 104: CALCULUS I

DATE: Monday, 16<sup>th</sup> July 2018

TIME: 11.00 a.m. - 1.00 p.m.

### INSTRUCTIONS:

#### QUESTION ONE (30 MARKS)

a) Use the definition of <sup>derivative</sup> ~~deviation~~ to compute  $f'(x)$  for  $f(x) = \frac{5}{x}$ . (5 Marks)

b) Evaluate the following limits.

i)  $\lim_{x \rightarrow \infty} \frac{5x^2 - x + 6}{x^2 + 9x + 7}$  (3 Marks)

ii)  $\lim_{y \rightarrow 1} \frac{y - 1}{\sqrt{y + 3} - 2}$  (4 Marks)

c) Find the value of A for which the following function is continuous.

$$f(x) = \begin{cases} x + 1, & x \leq 1 \\ 4 - Ax^2, & x > 1 \end{cases} \quad (4 \text{ Marks})$$

d) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^2 - 4$  and  $g(x) = x + 6$ , find

i)  $g \circ f$  and  $f \circ g$  (3 Marks)

ii)  $(f \circ g)^{-1}$  (2 Marks)

e) For the function implicitly represented by:

$$x^3 + 4y^3 - 3xy = 0, \text{ find}$$

i)  $\frac{dy}{dx}$  at the point (1,-1) (3 Marks)

ii) The equation of the tangent line to the curve at (1,-1). (2 Marks)

f) Verify that the function  $y = Ae^{kx} + Be^{-kx}$  satisfies the equation  $y'' - k^2 y = 0$ , where A, B, k are constant. (4 Marks)

### QUESTION TWO (20 MARKS)

a) For each of the following curves, find the gradient at the point with x - co-ordinate 2.

(i)  $y = \frac{3x}{2x+1}$  (3 Marks)

(ii)  $y = \sqrt{4x^2 + 9}$  (3 Marks)

b) Differentiate with respect to x

$y = x^3 \ln(5x+2)$  (2 Marks)

c) Find the derivatives of the following functions at the indicated point.

(i)  $\frac{(3x-8)^3}{x}, \quad x=3$  (3 Marks)

ii)  $\log\left(\frac{x^3}{\sqrt{x}}\right), \quad x=\frac{5}{2}$  (3 Marks)

iii)  $\cos x \cdot \log(3x), \quad x=\pi$  (3 Marks)

Question 5

(a) Let  $f(x) = \sqrt{x^2 - 2}$  and  $g(x) = 5x^2 + 1$

(i) Evaluate  $h(2)$  where  $h = g(f(x))$  (4 marks)

(ii) Find  $f^{-1}(g^{-1}(x))$  (5 marks)

(b) Let  $f_1(x) = \sqrt[4]{3-x}$ ,  $f_2(x) = \sqrt{x+1}$ . Find the domain of the functions:

(i)  $f_1$  and  $f_2$  (2 marks)

(ii)  $f_1 + f_2$  (2 marks)

(c) Find the equations of tangent and normal to a curve given parametrically  $x = \sqrt{t}$ ,  $y = \sqrt[3]{t}$  when  $t = 1$ . (6 marks)



Date: .....

KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2017/2018

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 104: CALCULUS 1

DATE: Thursday 23<sup>rd</sup> November, 2017

TIME: 2.00 p.m. - 4.00 p.m.

INSTRUCTIONS:

Question one [30 marks]

- a) Use the definition of derivative to compute  $f'(x)$  for  $f(x) = \frac{1}{2x}$ . [5 marks]
- b) Find  $\lim_{x \rightarrow \infty} \frac{(2x+4)^4}{(3x^2+1)^2}$  [3 marks]
- c) Find the value of  $k$  for which the following function is continuous [3 marks]
- $$f(x) = \begin{cases} 3x+2 & x < 2 \\ x^2+k & x \geq 2 \end{cases}$$
- d) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $g: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = \frac{1}{x-1}$ ,  $g(x) = \frac{x-1}{x+1}$ , determine  $f \circ g$ . [5 marks]
- e) Find the domain and the range of the function  $\frac{5x}{x^2-3x-4}$ . [4 marks]
- f) Verify whether the function  $y = A \cos 2x + B \sin 2x$  satisfies the equation  $y'' + 4y = 0$  [5 marks]
- g) For the function implicitly represented by  $x^5 + y^5 - 2xy = 0$ , find
- $\frac{dy}{dx}$  at the point  $(1,1)$ , [3 marks]
  - The equation of the tangent line to the curve at  $(1,1)$ . [2 marks]

a) i) Using the definition of continuity, prove that the function  $f(x) = 3x + 2$  is continuous at  $x = 2$ . [3 marks]

ii) Find the constants  $a$  and  $b$  so that the function

$$f(x) = \begin{cases} 2x & x \geq -1 \\ ax + b & -1 < x < 1 \\ x^2 + 3 & x \geq 1 \end{cases}$$

Is continuous on the entire real line.

[5 marks]

b) Find the derivative of the function  $y = \ln(\sqrt{2 \cos 2x}) + \frac{5}{x} + 2xe^{2x}$  [5 marks]

c) From first principles, differentiate the following function with respect to  $x$

$$y = \frac{1}{\sqrt{x}}$$

[7 marks]

**QUESTION THREE (20 MARKS)**

a) A particle moves on the hyperbola  $3x^2 - y^2 = 12$  in such a way that its  $y$  coordinate increases at a constant rate of 9 units per second. How fast is its  $x$  coordinate changing when  $x = 3$ . [5 marks]

b) Given that  $x = 2 \cos t$ ,  $y = 5 \sin t$  find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  [5 marks]

c) Sketch the curve  $y = x^3 - 3x + 3$ . [6 marks]

d) Find derivative of  $y = (x+5)^x$  [4 marks]

**QUESTION FOUR (20 MARKS)**

a) An open box is to be made from a 16 inch by 30 inch piece of card board by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume? [7 marks]

b) Differentiate the function  $f(x) = \cos x$  using first principles [6 marks]

c) Given the following functions  $f(x) = \sqrt{2x^2 - 4}$  and  $g(x) = 4x^2 - 2$ . Find  $f^{-1}(g^{-1}(x))$  [7 marks]



# KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF  
BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

## SMA 104: CALCULUS I

DATE: Monday, 8<sup>th</sup> May 2017

TIME: 8.00 a.m. - 10.00 a.m.

### INSTRUCTIONS:

Answer question ONE and any other TWO.

#### Question 1 (30 marks)

(a) Use the definition of derivative to compute  $f'(x)$  for  $f(x) = \frac{3}{x}$  (5 marks)

(b) Evaluate the following limits

(i)  $\lim_{x \rightarrow \infty} \frac{2x^2 - x + 3}{x^2 - 8x + 5}$  (3 marks)

(ii)  $\lim_{x \rightarrow 4^+} \frac{\sqrt{x+2} - 6}{x-4}$  (3 marks)

(c) Find the value of  $k$  for which the following function is continuous.

$$f(x) = \begin{cases} (x+k)^2 & : x < 3 \\ 5x+k & x \geq 3 \end{cases} \quad (4 \text{ marks})$$

(d) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $h: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 3x + 4$ ,  $h(x) = \frac{x}{5}$ , verify that

$$(h \circ f)^{-1} = f^{-1} \circ h^{-1} \quad (4 \text{ marks})$$

(e) Determine and state whether or not the function  $y = A \cos ax + B \sin ax$  satisfies the equation  $y'' + a^2 y = 0$  (3 marks)

(f) For the function implicitly represented by  $x^5 + y^5 - 2xy = 0$ , find

(i)  $\frac{dy}{dx}$  at the point  $(1, 1)$  (3 marks)

(ii) the equation of the tangent line to the curve at  $(1, 1)$  (2 marks)

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### Question 2

(a) Find the derivative of the following functions:

(i)  $y = (x^2 + 5x - 16)^{10}$  (3 marks)

(ii)  $y = \ln(e^x + 6 \sin x - 4 \sin^{-1} x)$  (4 marks)

(iii)  $y = \frac{x^8}{8(1-x^2)^4}$  (4 marks)

(b) (i) Find  $\frac{d^2y}{dx^2}$  for the function given in parametric form:

$x = \tan^{-1}(t), y = \ln(1 + t^2)$  (5 marks)

(ii) Verify that the function  $y = Ae^x + Be^{-x} - \frac{1}{x}$  satisfies the equation

$$y'' - y = \frac{x^2 - 2}{x^3}$$

### Question 3

(a) The concentration of an average student during a 3 hour test at time  $t$  is given by

$$c(t) = 2t^3 - 3t^2 - 12t + 30.$$

When in the test, is the students concentration maximal? (7 marks)

(b) Sketch the curve  $y = 2x^3 + 3x^2 - 3x + 12$ . (7 marks)

(c) Prove that the rectangle of a given parameter has maximum area when it is a square and express this maximum area in terms of the given parameter. (5 marks)

### Question 4

(a) (i) Using the definition of continuity, prove that the function  $f(x) = 3x + 2$  is continuous at  $x = 2$ . (3 marks)

(ii) Find the constants  $a$  and  $b$  so that the function

$$f(x) = \begin{cases} 2x & : x \leq -1 \\ ax + b & : |x| < 1 \\ x^2 + 3 & : x \geq 1 \end{cases}$$

is continuous on the entire real line. (5 marks)

(b) Differentiate the function with reference to  $x$

(i)  $y = 3x^2$  (3 marks)

(ii)  $y = \frac{2}{\sqrt{x}}$  (4 marks)

**Question Five (20 marks)**

- a) Consider the function  $f(x) = (x^2 - 1)^3$
- i) Find the intervals on which  $f$  is increasing or decreasing. (3 marks)
  - (ii) Find the local maximum and minimum values of  $f$ . (2 marks)
  - (iii) Find the intervals of concavity and the inflection points. (4 marks)
  - (iv) Use the information from parts (i), (ii) and (iii) to sketch the graph of  $f$ . (3 marks)

- b) Find the points where the given functions are not defined. For each such point state whether this discontinuity is removable.

(i)  $f(x) = \frac{x^3 - 27}{x^2 - 9}$  (3 marks)

(ii)  $f(x) = \begin{cases} 1 + x^2 & \text{if } x \leq 0 \\ \frac{\sin x}{x} & \text{if } x > 0 \end{cases}$  (2 marks)

(iii)  $f(x) = \frac{x - 17}{|x - 17|}$  (3 marks)

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- c) A box has a square base and the sum of its height and one side of the base is 20 cm. Find the maximum volume of the box. (6 marks)

- d) Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{2^x - 3^x}{x}$$

(3 marks)

#### Question Four (20 marks)

- a) Find the values of  $a$  and  $b$  that make  $f$  continuous.

$$f(x) = \begin{cases} 2x, & \text{if } x < 1 \\ ax + b, & \text{if } 1 \leq x \leq 2 \\ 4x, & \text{if } x > 2 \end{cases}$$

(5 marks)

- b) Evaluate the following limits

(i)  $\lim_{x \rightarrow -2} \frac{x+2}{x^2+x-2}$

(2 marks)

(ii)  $\lim_{x \rightarrow +\infty} \sqrt{x} - x$

(3 marks)

- c) Differentiate the function  $f(x) = x^{x/2}$  from first principles. (6 marks)

- d) Show that the derivative of  $y = \frac{2x}{\sqrt{3x^2+4}}$  simplifies to

$$\frac{dy}{dx} = \frac{8}{(3x^2+4)^{3/2}}$$

(4 marks)



## KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF  
BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

### SMA 104: CALCULUS I

DATE: Monday, 8<sup>th</sup> May 2017

TIME: 8.00 a.m. - 10.00 a.m.

#### INSTRUCTIONS:

Answer question ONE and any other TWO.

#### Question 1 (30 marks)

(a) Use the definition of derivative to compute  $f'(x)$  for  $f(x) = \frac{3}{x}$  (5 marks)

(b) Evaluate the following limits

(i)  $\lim_{x \rightarrow \infty} \frac{2x^2 - x + 3}{x^2 - 8x + 5}$  (3 marks)

(ii)  $\lim_{x \rightarrow 4^+} \frac{\sqrt{x+2} - 6}{x-4}$  (3 marks)

(c) Find the value of  $k$  for which the following function is continuous.

$$f(x) = \begin{cases} (x+k)^2 & : x < 3 \\ 5x+k & x \geq 3 \end{cases} \quad (4 \text{ marks})$$

(d) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $h: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 3x + 4$ ,  $h(x) = \frac{x}{5}$ , verify that

$$(h \circ f)^{-1} = f^{-1} \circ h^{-1} \quad (4 \text{ marks})$$

(e) Determine and state whether or not the function  $y = A \cos ax + B \sin ax$  satisfies the equation  $y'' + a^2 y = 0$  (3 marks)

(f) For the function implicitly represented by  $x^5 + y^5 - 2xy = 0$ , find

(i)  $\frac{dy}{dx}$  at the point  $(1, 1)$  (3 marks)

(ii) the equation of the tangent line to the curve at  $(1, 1)$  (2 marks)

### Question 2

(a) Find the derivative of the following functions:

(i)  $y = (x^2 + 5x - 16)^{10}$  (3 marks)

(ii)  $y = \ln(e^x + 6 \sin x - 4 \sin^{-1} x)$  (4 marks)

(iii)  $y = \frac{x^8}{8(1-x^2)^4}$  (4 marks)

(b) (i) Find  $\frac{d^2y}{dx^2}$  for the function given in parametric form:

$x = \tan^{-1}(t), y = \ln(1 + t^2)$  (5 marks)

(ii) Verify that the function  $y = Ae^x + Be^{-x} - \frac{1}{x}$  satisfies the equation

$$y'' - y = \frac{x^2 - 2}{x^3}$$

### Question 3

(a) The concentration of an average student during a 3 hour test at time  $t$  is given by

$$c(t) = 2t^3 - 3t^2 - 12t + 30.$$

When in the test, is the students concentration maximal? (7 marks)

(b) Sketch the curve  $y = 2x^3 + 3x^2 - 3x + 12$ . (7 marks)

(c) Prove that the rectangle of a given perimeter has maximum area when it is a square and express this maximum area in terms of the given parameter. (5 marks)

### Question 4

(a) (i) Using the definition of continuity, prove that the function  $f(x) = 3x + 2$  is continuous at  $x = 2$ . (3 marks)

(ii) Find the constants  $a$  and  $b$  so that the function

$$f(x) = \begin{cases} 2x & : x \leq -1 \\ ax + b & : |x| < 1 \\ x^2 + 3 & : x \geq 1 \end{cases}$$

is continuous on the entire real line. (5 marks)

(b) Differentiate the function with reference to  $x$

(i)  $y = 3x^2$  (3 marks)

(ii)  $y = \frac{2}{\sqrt{x}}$  (4 marks)

### Question 5

(a) Let  $f(x) = \sqrt{x^2 - 2}$  and  $g(x) = 5x^2 + 1$

(i) Evaluate  $h(2)$  where  $h = g(f(x))$  (4 marks)

(ii) Find  $f^{-1}(g^{-1}(x))$  (5 marks)

(b) Let  $f_1(x) = \sqrt[4]{3-x}$ ,  $f_2(x) = \sqrt{x+1}$ . Find the domain of the functions:

(i)  $f_1$  and  $f_2$  (2 marks)

(ii)  $f_1 + f_2$  (2 marks)

(c) Find the equations of tangent and normal to a curve given parametrically  $x = \sqrt{t}$ ,  $y = \sqrt[3]{t}$  when  $t = 1$ . (6 marks)

Success!  
@Jeff