# SMA1042011 0706 - Assignment

BUSINESS MANAGEMENT (Kenyatta University)

# KENYATTA UNIVERSITY
## UNIVERSITY EXAMINATIONS 2010/2011
**OPEN, DISTANCE AND E-LEARNING EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE**

**SMA 104: CALCULUS I**

**DATE:** Wednesday 6th July 2011 **TIME:** 2.00p.m – 4.00p.m

**INSTRUCTIONS: Answer Question ONE and any other TWO Questions.**

**QUESTION ONE (30MARKS)**

a) Evaluate the following limits.

i) $\lim_{x \to 1} \frac{x^2 + 6x - 7}{x - 1}$ [2]

ii) $\lim_{x \to 0} \frac{\tan x - x}{x^3}$ [3]

b) Use the First principle to find the derivative of the function
$y = \frac{1}{x^3}$ with respect to x. [5]

c) Use chain rule to differentiate
$y = h(x) = (x^2 - 6x + 5)^3$ [3]

d) Use product rule to differentiate
$y = h(x) = e^{2x^3}\left(x^2 + x + 2\right)$ [3]

e) Given the function $f(x) = x^4 - 2x^2 + 7$. Find the turning points. [5]

f) The position of a particle on a line is given by $s(t) = t^3 - 3t^2 - 6t + 5$, where $t$ is measured in seconds and $s$ is measured in feet. Find

i) The velocity of the particle at the end of 2 seconds. [2]

ii) The acceleration of the particle at the end of 2 seconds. [2]

g) Find the equation of the normal line to the parabola $y = 4x^2$ at the point $(-1,4)$ [5]

**QUESTION TWO (20MARKS)**

a) Find the derivative of $y = e^{\sqrt{x^3}+4}$ [3]

b) Find the equation of the tangent and normal at a point for which t = 2, given that the

parametric equations at a curve are $x = \dfrac{3t}{1+t}$, $y = \dfrac{t^2}{1+t}$ [7]

c) Find the turning points of the curve $y = x^4 - 6x^2 + 8x + 10$ distinguish them, hence sketch the curve. [10]

**QUESTION THREE (20MARKS)**

a) Use implicit differentiation to evaluate

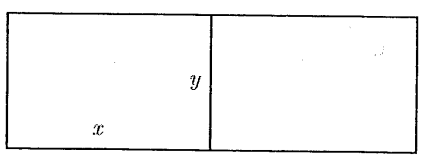
$$x^2 + y^2 - 2x - 6y + 5 = 0$$

[3]

b) Find $\dfrac{dy}{dx}$ given that $y = \sqrt{\dfrac{2x+1}{3x-2}}$ [6]

c) If $y = \sqrt{1+\sin x}$, show that $\dfrac{dy}{dx} = \dfrac{1}{2}\sqrt{1-\sin x}$ [5]

d) We have 1200 metres of fencing material, and wish to enclose a double paddock with two equal rectangular areas as shown in the diagram below.




Suppose that each of the two rectangular areas has sides x and y in metres, as shown in the picture. Find the dimensions of x and y that will maximize the area enclosed by the fence. [6]

**QUESTION FOUR (20MARKS)**

a) Find $\frac{dy}{dx}$ for the function $y = (x-1)\sqrt{x^2 - 2x + 2}$ [3]

b) Evaluate the limit

$$\lim_{x \to 3} \frac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$$ [5]

c) Gas is escaping from a spherical balloon at the rate of $900\text{cm}^3$/sec. How fast is the surface area shrinking when the radius is $360\text{cm}^2$? [5]

d) A closed rectangular container has a square base and is required to have a volume of $64\text{cm}^3$, if the container is made of thin metal find the dimensions which will minimize the surface area. [7]

**QUESTION FIVE (20MARKS)**

a) Find the derivative of the function $y=x^3\sin(3x+2)$. [3]

b) If $y = e^{3x}\sin 4x$. Find

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} + 25y$$ [7]

c) A rental agent estimates that the monthly profit p from a building 5 storeys high is given by $p = 4600s - 100s^2$. What height would maximize the profitable? [4]

d) Determine the area of the largest piece of rectangular ground that can be enclosed by 100m of fencing, if part of an existing wall is used as one side. [6]