SMA1042015 0428 - For assistance

INSTRUCTIONS:

Attempt question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

(a)   Using the precise definition of limits show that

$$\lim_{n \to \infty} \left\{ \frac{(-1)^n}{n} \right\} \to 0 \text{ where } n \in N.$$ Hence evaluate $N(\in)$ when

    (i) $\varepsilon = 0.02$
    (ii) $\varepsilon = 0.006$ **(4 marks)**

(b)   Explaining every step evaluate the following limits

    (i) $\text{Lim}_{n \to \infty} \dfrac{3n^2 - 4n + 9}{2n^2 + 5}$ **(2 marks)**

    (ii) $\lim_{x \to 0} \dfrac{\sqrt[m]{1 + ax} - \sqrt[n]{1 + bx}}{x}$ **(2 marks)**

    (iii) $\lim_{x \to \infty} \dfrac{(x - 3)^{40} (5x + 6)^{10}}{(3x^2 - 6)^{25}}$ **(2 marks)**

    (iii) $\lim_{x \to 0} \dfrac{x^x \sin 2x}{3x}$ **(2 marks)**

INVOLVEMENT IN ANY EXAMINATION IRREGULARITY SHALL LEAD TO DISCONTINUATION

(c)   From the first principles find $\frac{dy}{dx}$ of the following functions;

     (i) $f(x) = cosx$ **(3 marks)**

     (ii) $f(x) = \sqrt{x+2}$ **(3 marks)**

(d)   Let $\lim_{n \to \infty} X_n \to A$ and $\lim_{n \to \infty} y_n \to B$ Show that

     (i) $\lim_{n \to \infty} x_n + y_n = A + B$ **(4 marks)**

     (ii) $\lim_{n \to \infty} x_n \, y_n = A \cdot B$ **(4 marks)**

   (e) Deduce the quotient rule of differentiation   **(4 marks)**

QUESTION TWO (20 MARKS)

(a)   Given $y = \frac{\cos x}{x}$, hence or otherwise prove that $x \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$

                                                               (4 marks)

b)   For what value of the constant B is the following function continuous for all $x \in \Re$

    i)F(x) = $\begin{cases} X^2 - 3x + 4 & \text{if } x > 1 \\ \\ Bx + 5 & \text{if } x < 1 \end{cases}$

    ii) F(x) = $\begin{cases} 5X + B & \text{if } x > 3 \\ \\ x + B)^2 & \text{if } x < 3 \end{cases}$

                                                                (6marks)

(c) let f(x)=$\sqrt{x}$, g(x)=4x-8 and h(x)=2x find fogoh and hence find(fogoh)$^{-1}$ **(2marks)**

(d) Determine whether $y = Ae^{ax} + Be^{-ax}$ is satisfied by $y'' - a^2y = 0$ **(3 marks)**

(e) Find the equation of the curve given the gradient is $4x - 2$ and $x - $ axis is the tangent.

                                                                 (5 marks)

QUESTION THREE (20 MARKS)

(a) A particle moves along a straight line in such away that its distance from a fixed point on the line after t seconds is S meters, where $S = \frac{1}{6}t^3$. Find;

    (i) Its velocity after 2 seconds and 3 seconds

    (ii) Its acceleration after 1 seconds and 4 seconds   **(5 marks)**

(b) A ball was thrown upwards with a velocity of 40 m/s. Find

    (i) The acceleration, velocity and distance statements.

    (ii) The maximum height the ball can attain (strictly use calculus techniques)   **(6 marks)**

(c) The volume of a cylindrical tank is $32\pi mls$. Find the radius of the tank if the area have to be least.   **(5 marks)**

(d) A 525 meter wire mesh was provided to fence a rectangular (like) plot. Find the maximum area it can enclose by mesh without any loss.   **(4 marks)**

QUESTION FOUR (20 MARKS)

(a) Use the first principle to find $\frac{dy}{dx}$ of the following functions

    (i) $y = lnx$ **(4 marks)**

    (ii) $y = \cos x$ **(4 marks)**

(b) Given $y = 2x^3 - 15x^2 + 24x + 19$ find the stationary points.   **(4 marks)**

(c) Differentiate the following functions.

    (i) $y = Sin^3 2x$ **(2 marks)**

    (ii) $y = \frac{Tan^2 e^x \{\cos x\}}{x^2}$ **(2 marks)**

(d) Evaluate (i) $Lim_{x \to \infty} \left\{ \frac{(3x + 1)}{3x - 2} \right\}^{2x}$ **(2 marks)**

    (ii) $Lim_{x \to 0} \frac{1 - \cos 3x}{x^2}$ **(2 marks)**

QUESTION FIVE (20 MARKS)

(a) Find y' and y'' given $y = \dfrac{e^{ax} + e^{-ax}}{e^{ax} - e^{-ax}}$ (6 marks)

(b) Find y'' given $y = e^{-2x} \sin 4x$ (5 marks)

(c) Find y' and y'' given $x^4 + xy^3 + y^3 = 32$ at the point $(1, 1)$ (5 marks)

(d) Show that $\lim_{x \to 2} x^2 + 2x + 8 = 16$ (4 marks)