

1503/203  
MATHEMATICS II  
Oct./Nov. 2021  
Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL  
CRAFT CERTIFICATE IN AUTOMOTIVE ENGINEERING  
MODULE II  
MATHEMATICS II

3 hours

**INSTRUCTIONS TO CANDIDATES**

*You should have the following for this examination*

*Answer booklet;*

*Scientific calculator;*

*Drawing instruments.*

*This paper consists of EIGHT questions.*

*Answer FIVE questions in the answer booklet provided.*

*All questions carry equal marks.*

*Maximum marks for each part of a question are as shown.*

*Candidates should answer the questions in English.*

**This paper consists of 4 printed pages.**

**Candidates should check the question paper to ascertain that  
all the pages are printed as indicated and that no questions are missing.**

1. (a) (i) Use the binomial theorem to expand

$$\sqrt{\frac{1+5x}{1-5x}}$$
 as far as the term in  $x^3$ .

- (ii) By setting  $x = \frac{1}{20}$ , determine the approximate value of  $\sqrt{15}$  correct to 5 decimal places.

(10 marks)

- (b) Use elimination method to solve the equations

$$3x + 5y + 2z = 38$$

$$6x - 2y + 5z = 34$$

$$4x + 3y + z = 26$$

(10 marks)

2. (a) Evaluate the integrals:

(i)  $\int_0^1 \left[ \frac{x(3x^2 + 5x + 7)}{x^4} \right] dx$

(ii)  $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$  correct to 3 decimal places

(10 marks)

- (b) Determine the area bounded by the curve

$$y = 6x - x^2, \text{ the } x\text{-axis and the line } y = 3x - 4.$$

(10 marks)

3. (a) Determine the angle between the vectors

$$A = 2\mathbf{i} + 4\mathbf{j} + 5\mathbf{k} \text{ and } B = 4\mathbf{i} + 6\mathbf{j} - 7\mathbf{k}.$$

(6 marks)

- (b) Given the scalar field  $\phi(x, y, z) = x^4 + y^4 + z^4$ . Determine the directional derivative of  $\phi$  at the point  $A(1, -2, 1)$  in the direction of  $AB$ , where  $B$  is  $(2, 6, -1)$ . (7 marks)

- (c) A particle's displacement at any time  $t$  is given by  $x = 2t^3 - 4t$ ,  $y = 5t^2 + 4t$  and  $z = 8t^2 - 3t^3$ . Determine at  $t = 2$  the:

- (i) velocity;  
(ii) acceleration of the particle.

(7 marks)



4. (a) Given the matrix  $A = \begin{bmatrix} 6 & -6 & -12 \\ -3 & 9 & 12 \\ 3 & -6 & -9 \end{bmatrix}$ , determine  $B = A^3 - 2A^2$ . (8 marks)

(b) Use the inverse matrix method to solve the equations:

$$4x + 5y - 3z = 4$$

$$3x + 2y + z = 14$$

$$-5x + y + z = 3$$

(12 marks)

5. (a) Given that  $V(x, y) = (x^3 + y^3) - 4Mx^2y^3$  where  $M$  is a constant, show that

$$\frac{x^2 \partial^2 V}{\partial x^2} - y^2 \frac{\partial^2 V}{\partial y^2} = 6(x^3 - y^3). \quad (6 \text{ marks})$$

(b) The radius of a cone decreases by 4% while the height increases by 5%. Use partial differentiation to determine the change in volume. (5 marks)

(c) Locate stationary points of the function  $f(x, y) = 2x^2 + 7xy + 5y^2 - 69x - 105y + 21$  and determine their nature. (9 marks)

6. (a) Given that  $\cos \theta = \frac{5}{13}$ , determine the other five trigonometric ratios of  $\theta$ . (6 marks)

(b) Prove the identity:

$$\frac{\sin \theta}{1 + \sec \theta} - \frac{\sin \theta}{1 - \sec \theta} = 2 \cot \theta. \quad (6 \text{ marks})$$

(c) Solve the equation:  $2 \sin^2 \theta = -3 \cos \theta$  for values of  $\theta$  between  $0^\circ$  and  $360^\circ$  inclusive. (8 marks)

7. (a) Determine the values of  $S$  and  $R$  such that  $R \cosh 2x + S \sinh 2x = 4e^{2x} + 7e^{-2x}$ . (5 marks)

(b) Determine the logarithmic form of  $\tanh^{-1} x$ . (7 marks)

(c) Solve the equation:

$$2 \sinh 2x + 3 \cosh 2x = 4. \quad (8 \text{ marks})$$

8. (a) Find  $\frac{dy}{dx}$  of the function  $y = \sin x$  from first principles. (6 marks)
- (b) Use implicit differentiation to determine the equation of the normal to the function  $x^3 + 2xy + 5y^2 = 8$  at the point (1, 1). (7 marks)
- (c) A particle moves from rest to the point R such that its displacement is given by  $S = \frac{t^3}{3} + \frac{7}{2}t^2 + 12t + 8$ . Determine:
- (i) the displacement at  $t = 2$ .
  - (ii) the velocity at  $t = 2$ .
  - (iii) when its momentarily at rest.
- (7 marks)

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